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Excitation and propagation of Bleustein-Gulyaev waves

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TABLE OF CONTENTS

I. INTRODUCTION

In recent years piezoelectric surface waves have been the subject of a considerable amount of research. References [1,2] provide typical reviews and a comprehensive bibliography of the field. This thesis treats a number of problems associated with a special type of surface wave that is sufficiently new to bear the burden of at least three different names; the Bleustein mode, a Bleustein-Gulyaev wave, and an S-H (shear-horizontally polarized) mode. This is a free surface disturbance in which the elastic displacement is normal to the sagittal plane[1] and the electric field is parallel to the sagittal plane.

The wave described above has no elastic counterpart; the associated electric field is necessary in order to satisfy the free surface boundary conditions. Its existence was proven independently by Bleustein [3] and Gulyaev [4]. In both works the proof was confined to the case where the substrate was assumed to have 6mm symmetry, the surface was cut parallel to the zonal axis, and the propagation direction was normal to the zonal axis. Koerber [5] showed that the wave equations and the boundary condition equations decouple in such a fashion that the Bleustein-Gulyaev wave has potential existence for any cut and propagation direction where the sagittal plane is normal to a 2-fold axis of symmetry. Tseng [6] and Koerber and Vogel [7] have proven the existence of this mode in a wide variety of cases other than those treated by Bleustein and Gulyaev.

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[1] The term sagittal plane is described in the next section.

This thesis is confined to the former case or its equivalent, i.e., substrates having 6mm, 4mm, 6, or 4 symmetry. There are two reasons for this. First, there are a large number of practical piezoelectric materials, including such familiar cases as ZnO, CdS, CdSe, and the ferroelectric ceramics, that fall into this category. Second, the analysis of these cases is simplified because they all are transversely isotropic, i.e., all of the relevant tensor properties are invariant under an arbitrary rotation about the zonal axis.

Heretofore, analysis of the properties of the Bleustein-Gulyaev wave have been confined to cases where the disturbance is propagating upon an infinite plane interface with a remote source. This thesis provides the theoretical basis for generalizing these analyses in two ways; it deals with the propagation of a Bleustein-Gulyaev wave around cylindrical surfaces, and it deals with excitation. In both cases specific problems are developed to completion, and a number of elements of physics pertinent to this disturbance are deduced.

II. BLEUSTEIN-GULYAEV WAVES ON A FLAT SURFACE

A. Electrodynamic Maxwell's Equations

1. Free space interface

As in all piezoelectric wave problems solved in this study, the equations to be considered are the elastic equation of motion, Maxwell's equations, and the constitutive relations. After elastic and electromagnetic wave forms are found satisfying these equations in the piezoelectric medium and in the exterior free space, the waves are coupled at the interface through the boundary conditions. In this section we will go through the analysis of a surface wave on a flat surface in detail so that later sections, where the analysis is similar, may be condensed.

Figure 1 shows the coordinates and orientation of the elastic and electric fields. The piezoelectric medium occupies the regions $x_2 \geq 0$ and the region $x_2 < 0$ is free space. The surface wave propagates in the x_1 direction with its elastic displacement in the x_3 direction and electric fields in the sagittal plane (the plane parallel to the direction of propagation and the normal to the surface). The zonal axis of the piezoelectric medium is parallel to the x_3 axis. Throughout this study the external electric field will be denoted by a circumflex.

Since u_3 is the only non-zero elastic displacement component, the elastic equation of motion, which is derived from Newton's second principle, is

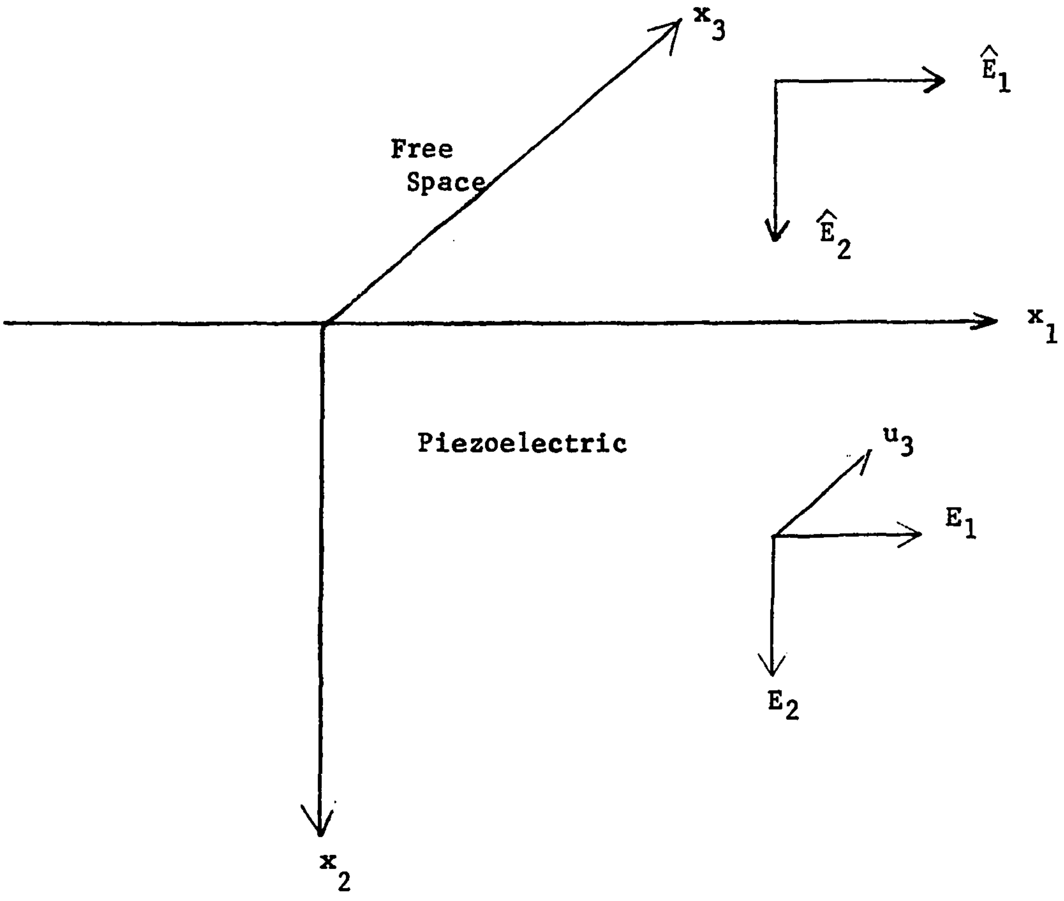

Figure 1. Surface wave geometry

$$\rho \frac{\partial^2 u_3}{\partial t^2} = \frac{\partial T_{3j}}{\partial x_j} \tag{1}$$

where ρ is the density of the piezoelectric and T_{3j} is the stress tensor.

The constitutive equations, which relate the stress, strain, electric flux, and electric intensity fields in the medium, for 6mm symmetry are

$$T_{23} = c_{44} \frac{\partial u_3}{\partial x_2} - e_{15}E_2 \tag{2a}$$

$$T_{13} = c_{44} \frac{\partial u_3}{\partial x_1} - e_{15}E_1 \tag{2b}$$

$$D_2 = e_{15} \frac{\partial u_3}{\partial x_2} + \epsilon_{11}E_2 \tag{2c}$$

$$D_1 = e_{15} \frac{\partial u_3}{\partial x_1} + \epsilon_{11}E_1 \tag{2d}$$

where D_i is the electric flux density, E_i is the electric field intensity, and c_{44}, e_{15}, and ϵ_{11} are the elastic, piezoelectric, and permittivity tensors of the piezoelectric. From here on we will delete the subscripts on the material tensors and on u_3. In the exterior free space the constitutive relation is

$$\hat{\bar{D}} = \epsilon_o \hat{\bar{E}} \quad . \tag{3}$$

The electrodynamic Maxwell's equations are

$$\bar{\nabla} \times \bar{E} = -\mu_o \frac{\partial \bar{H}}{\partial t} \tag{4a}$$

$$\bar{\nabla} \times \bar{H} = \frac{\partial \bar{D}}{\partial t} \tag{4b}$$

$$\bar{\nabla} \cdot \bar{D} = 0 \tag{4c}$$

where the permeability of the piezoelectric medium, μ_o, differs insignificantly from the permeability of free space. The magnetic flux equation, $\bar{\nabla} \cdot \bar{B} = 0$, does not enter this study.

Having described the significant equations in this problem, let us express the electric field intensity as a sum of an irrotational field and a solenoidal field,

$$\bar{E} = \bar{E}^i + \bar{E}^s = - \bar{\nabla}\phi - \bar{\nabla} \times \bar{F} \tag{5}$$

where the superscripts "i" and "s" denote irrotational and solenoidal fields respectively, ϕ is a scalar electric potential, and $\bar{F}$ is a vector electric potential. Since $\bar{E}$ is in the $x_1 - x_2$ plane let $\bar{F}$ be parallel to the x_3 axis,

$$\bar{F} = \bar{n}\,\psi \tag{6}$$

where $\bar{n}$ is a unit vector in the x_3 direction.

The substitution of the constitutive relations 2c and 2d into Maxwell's equation 4c, with the electric fields expressed in terms of the electric potentials 5 and 6, leads to an equation relating the elastic displacement and the irrotational electric field,

$$e\nabla^2 u - \epsilon\nabla^2\phi = 0 \tag{7}$$

where ∇^2 is the two-dimensional Laplacian operator. This equation is identically zero if

$$\phi = \frac{e}{\epsilon} u \; . \tag{8}$$

Substitution of Eqs. 5, 6, and 8 into the constitutive relations 2 leads to a new form for the constitutive relations,

$$T_{23} = \bar{c}\,\frac{\partial u}{\partial x_2} - e\,\frac{\partial \psi}{\partial x_1} \tag{9a}$$

$$T_{13} = \bar{c}\,\frac{\partial u}{\partial x_1} + e\,\frac{\partial \psi}{\partial x_2} \tag{9b}$$

$$D_2 = \epsilon\,\frac{\partial \psi}{\partial x_1} \tag{9c}$$

$$D_1 = -\,\epsilon\,\frac{\partial \psi}{\partial x_2} \tag{9d}$$

where $\bar{c} = c + \frac{e^2}{\epsilon}$ is a "stiffened" elastic constant.

Since the electric flux density is a function of only the solenoidal electric field, manipulation of Eqs. 9c and 9d and Maxwell's equations 4a and 4b leads to the familiar Helmholtz wave equation

$$\nabla^2\psi + k_b^2\psi = 0 \tag{10}$$

where $k_b = \omega\sqrt{\mu_o\epsilon}$ is the propagation constant of a uniform plane electromagnetic wave in a dielectric of permittivity ϵ. Throughout this work we will assume $e^{-j\omega t}$ time dependence.

Likewise, when the stress constitutive relations 9a and 9b are substituted into the elastic equation of motion, Eq. 1, terms involving the irrotational electric field cancel. The result is a Helmholtz equation for the elastic wave,

$$\nabla^2 u + k_a^2 u = 0 \tag{11}$$

where $k_a = \omega\sqrt{\frac{\rho}{\bar{c}}}$ is the propagation constant for a uniform plane shear wave.

If outside the piezoelectric we let

$$\hat{E} = -\,\bar{\nabla} \times \bar{n}\,\hat{\psi} \tag{12}$$

Maxwell's equations and the constitutive relations 3 reduce to

$$\nabla^2 \hat{\psi} + k_c^2 \hat{\psi} = 0 \tag{13}$$

where $k_c = \omega \sqrt{\mu_o \varepsilon}$ and

$$D_2 = \varepsilon_o \frac{\partial \hat{\psi}}{\partial x_1} \tag{14a}$$

$$D_1 = - \varepsilon_o \frac{\partial \hat{\psi}}{\partial x_2} \; . \tag{14b}$$

Wave forms satisfying Eqs. 10, 11, and 13 are

$$u = Ae^{j \sqrt{k_a^2 - k_1^2}\, x_2 + jk_1 x_1} \tag{15a}$$

$$\psi = Be^{j \sqrt{k_b^2 - k_1^2}\, x_2 + jk_1 x_1} \tag{15b}$$

$$\hat{\psi} = Ce^{j \sqrt{k_c^2 - k_1^2}\, x_2 + jk_1 x_1} \tag{15c}$$

where A, B, and C are wave amplitudes and k_1 is the propagation constant in the x_1 direction. These are uniform plane waves if k_1 is less than k_a (and therefore less than k_b and k_c since the elastic wave velocity is much less than the electromagnetic wave velocity). If k_1 is greater than k_a the waves are "inhomogeneous" waves decaying exponentially in the plus or minus x_2 direction and propagating in the x_1 direction at velocity $v = \frac{\omega}{k_1}$. Obviously a surface wave is an inhomogeneous wave.

At this point let us consider the propagation of uniform plane waves in an unbounded piezoelectric medium. Since the electrodynamic wave equation 10 and the elastic wave equation 11 are not coupled, the electromagnetic wave and elastic wave propagate independently. Coupled to the elastic wave is an electrostatic field given by Eqs. 5 and 8,

$$\bar{E}^{i} = - \frac{e}{\varepsilon} \bar{\nabla} u \ .$$

The effect of this field is to increase or "stiffen" the elastic constant,

$$\bar{c} = c + \frac{e^2}{\varepsilon} \ .$$

This increases the elastic wave velocity,

$$v = \frac{\omega}{k_a} = \sqrt{\frac{\bar{c}}{\rho}}$$

over the velocity if the medium were not piezoelectric,

$$v = \sqrt{\frac{c}{\rho}} \ .$$

The solenoidal electromagnetic wave propagates through the piezoelectric at a velocity

$$v = \frac{\omega}{k_b} = \frac{1}{\sqrt{\mu_o \varepsilon}}$$

which is equal to the velocity in a purely dielectric medium. In addition a stress wave given by Eqs. 9a and 9b,

$$T_{23} = - e \frac{\partial \psi}{\partial x_1}$$

$$T_{13} = e \frac{\partial \psi}{\partial x_2}$$

is coupled to the electromagnetic wave. The stress wave produces no elastic strain and does not affect the velocity of the electromagnetic wave. This does not occur for all symmetry classes. In the general case the elastic and solenoidal electric fields are coupled. However, for the symmetry considered here, the elastic and solenoidal electric fields are coupled only at the boundary of the piezoelectric as we see below.

Rewriting Eqs. 14 in the "inhomogeneous" form gives

$$u = Ae^{-\sqrt{k_1^2-k_a^2}\,x_2 + jk_1x_1} \tag{16a}$$

$$\psi = Be^{-\sqrt{k_1^2-k_b^2}\,x_2 + jk_1x_1} \tag{16b}$$

$$\hat{\psi} = Ce^{\sqrt{k_1^2-k_c^2}\,x_2 + jk_1x_1} \tag{16c}$$

where the signs on the radicals are chosen to insure decay away from the surface. From here on we use the positive value of the radical.

To find the wave amplitudes B and C in terms of A and the surface wave propagation constant k_1 we satisfy the boundary conditions on the surface $x_2 = 0$. These conditions are zero stress,

$$T_{23} = 0 \qquad \text{at} \quad x_2 = 0 \tag{17a}$$

continuous tangential electric field intensity,

$$E_1 = \hat{E}_1 \qquad \text{at} \quad x_2 = 0 \tag{17b}$$

and continuous normal electric flux density,

$$D_2 = \hat{D}_2 \qquad \text{at} \quad x_2 = 0 \,. \tag{17c}$$

From the constitutive relation 9a the zero stress condition is

$$\bar{c}\,\frac{\partial u}{\partial x_2} - e\,\frac{\partial \psi}{\partial x_1} = 0 \qquad \text{at} \quad x_2 = 0 \,. \tag{18a}$$

From potential equations 5, 6, 8, and 12 the electric intensity condition is

$$-\frac{e}{\varepsilon}\,\frac{\partial u}{\partial x_1} - \frac{\partial \psi}{\partial x_2} = -\frac{\partial \hat{\psi}}{\partial x_2} \qquad \text{at} \quad x_2 = 0 \tag{18b}$$

and from the constitutive relations 9c and 14a, the flux density condition is

$$\varepsilon\,\frac{\partial \psi}{\partial x_1} = \varepsilon_o\,\frac{\partial \hat{\psi}}{\partial x_1} \qquad \text{at} \quad x_2 = 0 \,. \tag{18c}$$

Substituting wave forms 16 into the boundary conditions 18 leads to the following wave amplitude ratios and propagation constant secular equation:

$$\frac{B}{A} = \frac{j\bar{c}\,\sqrt{k_1^2 - k_a^2}}{e k_1} \tag{19a}$$

$$\frac{C}{B} = \frac{\varepsilon}{\varepsilon_o} \tag{19b}$$

$$K^2 k_1^2 - \sqrt{k_1^2 - k_a^2}\,\left(\sqrt{k_1^2 - k_b^2} + \frac{\varepsilon}{\varepsilon_o}\,\sqrt{k_1^2 - k_c^2} \right) = 0 \tag{19c}$$

where $K^2 = \frac{e^2}{\varepsilon c}$ is a dimensionless piezoelectric coupling constant.

Solving the secular equation for the surface wave propagation constant k_1 completes the solution. Since the secular equation is transcendental, an approximation that gives the propagation constant explicitly is desirable. This is derived in the following manner.

Since the elastic wave velocity is from 10^{-4} to 10^{-5} times less than the electromagnetic wave velocity, we have

$$\frac{k_b^2}{k_1^2} \text{ and } \frac{k_c^2}{k_1^2} \approx 10^{-8} \text{ to } 10^{-10}$$

(This ratio of the square of the acoustic velocity to electromagnetic velocity will often appear in this study.) Therefore, extremely good approximations for the electromagnetic radicals in the secular equations are

$$\sqrt{k_1^2-k_b^2} \approx k_1\left(1 - \frac{k_b^2}{2k_1^2}\right) \tag{20a}$$

$$\sqrt{k_1^2-k_c^2} \approx k_1\left(1 - \frac{k_c^2}{2k_1^2}\right) \tag{20b}$$

Squaring the secular equation, using these approximations, and solving for k_1^2 by the quadratic formula to the first order of $\frac{k_b^2}{k_a^2}$ leads to

$$k_1 \approx \frac{k_a}{\left[1 - \frac{K^4}{(1+\frac{\varepsilon}{\varepsilon_o})^2}\right]^{\frac{1}{2}}} \left[1 + \frac{K^4\, k_b^2}{(1+\frac{\varepsilon}{\varepsilon_o})^3 k_a^2}\right] \tag{21}$$

As the coupling constant K^2 goes to zero, k_1 approaches k_a and the surface wave degenerates to a uniform plane elastic wave propagating parallel to the surface.

In a later section it is shown that the first term in Eq. 21 is the "electrostatic approximation" solution. But first let us look at the surface wave when the interface is covered by an electric conductor.

2. Conducting interface

A very thin electric conductor at the interface (See Figure 1) will not load the surface elastically but will short the tangential electric field at the surface. Therefore, the stress boundary condition remains the same as in the free space problem, but the electric boundary conditions become

$$E_1 = 0 \qquad \text{at} \quad x_2 = 0 \tag{22a}$$

$$D_2 = \sigma \qquad \text{at} \quad x_2 = 0 \tag{22b}$$

where σ is the surface charge density on the conductor. Eq. 22b poses no additional constraint on the surface wave, but since zero external electric field means one less unknown parameter, one less boundary condition is needed to solve the problem. From Eqs. 18a and 18b the boundary conditions are

$$\bar{c}\,\frac{\partial u}{\partial x_2} - e\,\frac{\partial \psi}{\partial x_1} = 0 \qquad \text{at} \quad x_2 = 0 \tag{23a}$$

$$-\,\frac{e}{\epsilon}\,\frac{\partial u}{\partial x_1} - \frac{\partial \psi}{\partial x_2} = 0 \qquad \text{at} \quad x_2 = 0 \tag{23b}$$

The elastic and electromagnetic wave forms in the piezoelectric, Eqs. 16a and 16b, are rewritten here for reference:

$$u = Ae^{-\sqrt{k_1^2-k_a^2}\, x_2 + jk_1x_1}$$

$$\psi = Be^{-\sqrt{k_1^2-k_b^2}\, x_2 + jk_1x_1}$$

Substituting these wave forms into boundary conditions 23 leads to the following wave amplitude ratio and the propagation constant secuiar equation:

$$\frac{B}{A} = \frac{j\bar{c}\sqrt{k_1^2-k_a^2}}{ek_1} \tag{24a}$$

$$K^2k_1^2 - \sqrt{k_1^2-k_a^2}\;\sqrt{k_1^2-k_b^2} = 0 \tag{24b}$$

Squaring the secular equation and solving for k_1 to the first order of $\frac{k_b^2}{k_a^2}$ leads to a very good approximation for the propagation constant for a conducting interface;

$$k_1 \approx \frac{k_a}{[1 - K^4]^{\frac{1}{2}}}\,[1 + \frac{K^4k_b^2}{2k_a^2}] \tag{25}$$

In closing this section we note that solutions 24 for the conducting interface can be derived directly from solutions 19 for a free space interface by letting the free space permittivity, ϵ_0, approach infinity. This occurs because a medium with a high permittivity will short the electric field.

B. Electrostatic Maxwell's Equations

1. Free space interface

The electrostatic (or quasistatic) approximation is often used to simplify piezoelectric wave problems. In fact Bleustein used this approximation in his original study showing the existence of the surface wave named after him [3]. When the elastic wavelength is much shorter than the electromagnetic wave length at the same frequency, the electrostatic approximation may be valid. In this section it will be shown that this is a valid approximation for a piezoelectric surface wave on a flat surface. The derivation of the electrostatic surface wave solutions is similar to Bleustein's derivation.

In the electrostatic approximation Maxwell's equation 4a becomes

$$\bar{\nabla} \times \bar{E} = 0 \tag{26}$$

This is an identity if the electric field intensity is expressed as a vector gradient of a scalar electric potential,

$$\bar{E} = - \bar{\nabla} \Phi \tag{27}$$

in the piezoelectric medium and

$$\hat{\bar{E}} = - \bar{\nabla} \hat{\psi} \tag{28}$$

in the exterior free space. Substituting Eq. 27 and the electric flux density constitutive relations 2c and 2d into

$$\bar{\nabla} \cdot \bar{D} = 0 \tag{29}$$

leads to

$$e\nabla^2 u - \epsilon\nabla^2 \Phi = 0 \tag{30}$$

This is identically zero if we separate the electric potential in two potentials

$$\Phi = \phi + \psi \tag{31}$$

where ϕ is proportional to the elastic field

$$\phi = \frac{e}{\epsilon} u \tag{32}$$

and ψ is, at this point, an arbitrary potential satisfying Laplace's equation

$$\nabla^2 \psi = 0 \ . \tag{33}$$

In free space Eqs. 28 and 29 lead to

$$\nabla^2 \hat{\psi} = 0 \ . \tag{34}$$

Substituting Eqs. 27 and 31 into the constitutive relations 2 leads to

$$T_{23} = \bar{c} \frac{\partial u}{\partial x_2} + e \frac{\partial \psi}{\partial x_2} \tag{35a}$$

$$T_{13} = \bar{c} \frac{\partial u}{\partial x_1} + e \frac{\partial \psi}{\partial x_1} \tag{35b}$$

$$D_2 = - \epsilon \frac{\partial \psi}{\partial x_2} \tag{35c}$$

$$D_1 = - \epsilon \frac{\partial \psi}{\partial x_1} \tag{35d}$$

Substituting the stress equations 35a and 35b into the elastic equation of motion 1 leads to an elastic wave equation

$$\nabla^2 u + k_a^2 u = 0 \tag{36}$$

This is the same as the elastic wave equation 11 derived in the electrodynamic analysis. Relation 32 between the electrostatic field and the elastic displacement is also the same as in the electrodynamic analysis, Eq. 8. The difference is that in the electrodynamic analysis ψ is a solution of Helmholz's equation while in the electrostatic analysis ψ is a solution of Laplace's equation.

Note that if we let the electromagnetic propagation constant k_b and k_c go to zero, the Helmholtz equation degenerates to Laplace's equation. We can say that in the electrostatic limit the velocity of electromagnetic radiation, $v = \frac{\omega}{k_b}, \frac{\omega}{k_c}$, becomes infinite, which is just another way of describing the electrostatic approximation.

Waveforms satisfying Laplace's equations 33 and 34 and decaying away from the surface are

$$\psi = Be^{-k_1 x_2 + jk_1 x_1} \tag{37a}$$

$$\hat{\psi} = Ce^{k_1 x_2 + jk_1 x_1} \tag{37b}$$

The elastic wave form is the same as Eq. 16a

$$u = Ae^{-\sqrt{k_1^2 - k_a^2}\, x_2 + jk_1 x_1} \tag{37c}$$

From Eqs. 27, 28, 35a, and 35c the boundary conditions of vanishing stress, continuous electric field intensity, and continuous normal flux density at the interface are

$$\bar{c}\,\frac{\partial u}{\partial x_2} + e\,\frac{\partial \psi}{\partial x_2} = 0 \qquad \text{at} \quad x_2 = 0 \tag{38a}$$

$$-\frac{e}{\epsilon}\,\frac{\partial u}{\partial x_1} - \frac{\partial \psi}{\partial x_1} = -\frac{\partial \hat{\psi}}{\partial x_1} \qquad \text{at} \quad x_2 = 0 \tag{38b}$$

$$-\epsilon\,\frac{\partial \psi}{\partial x_2} = -\epsilon_o\,\frac{\partial \hat{\psi}}{\partial x_2} \qquad \text{at} \quad x_2 = 0 \tag{38c}$$

Substituting the wave forms 37 into Eqs. 38 leads to the following electrostatic approximation solutions for the surface wave:

$$\frac{B}{A} = -\frac{e}{\epsilon\left(1 + \frac{\epsilon}{\epsilon_o}\right)} \tag{39a}$$

$$\frac{C}{B} = -\frac{\epsilon}{\epsilon_o} \tag{39b}$$

$$k_1 = \frac{k_a}{\left[1 - \frac{K^4}{\left(1 + \frac{\epsilon}{\epsilon_o}\right)^2}\right]^{\frac{1}{2}}} \tag{39c}$$

These are Bleustein's solutions. Note that Eq. 39c is an explicit expression for k_1.

This is a good opportunity to compare the electrostatic and electrodynamic solutions. Comparing Eq. 39c to the electrodynamic expansion, Eq. 21, shows that the electrostatic solution is the first term in the electrodynamic expansion. Since the second term is of the order of 10^{-8} or 10^{-10} the size of the first term, or even less depending on the magnitude of $\frac{K^4}{\left(1 + \frac{\epsilon}{\epsilon_o}\right)^3}$, the electrostatic approximation is an extremely accurate approximation. It gives a propagation constant slightly smaller than the exact propagation constant or a velocity slightly larger than the exact velocity.

2. Conducting interface

When the interface is covered by a very thin conductor the boundary conditions, Eqs. 38a and 38b, become

$$\bar{c}\,\frac{\partial u}{\partial x_2} + e\,\frac{\partial \psi}{\partial x_2} = 0 \qquad \text{at} \quad x_2 = 0 \tag{40a}$$

$$-\,\frac{e}{\epsilon}\,\frac{\partial u}{\partial x_1} - \frac{\partial \psi}{\partial x_1} = 0 \qquad \text{at} \quad x_2 = 0 \tag{40b}$$

Substituting waveforms 37a and 37c into Eq. 40 gives the following amplitude ratio and propagation constant:

$$\frac{B}{A} = -\,\frac{e}{\epsilon} \tag{41a}$$

$$k_1 = \frac{k_a}{\sqrt{1 - K^4}} \tag{41b}$$

Eq. 41b is the first term in the electrodynamic expansion, Eq. 25. Again the electrostatic approximation gives a propagation constant slightly smaller than the exact propagation constant.

III. BLEUSTEIN-GULYAEV WAVES ON A CYLINDRICAL SURFACE

A. Propagation Around a Solid Cylinder

1. Free space interface

This chapter analyzes the propagation of Bleustein-Gulyaev waves around a piezoelectric solid cylinder. To the author's knowledge no exact piezoelectric cylindrical wave solution has been published previously.

Figure 2 shows the geometrical configuration for this problem. The zonal axis of the piezoelectric medium is parallel to the cylinder axis. In the crystal classes studied here, the material tensors are independent of a rotation of the reference axis around the zonal axis. It is this "transverse isotropy" that makes the surface wave problem tractable. The elastic displacement is parallel to the cylinder axis and the electric fields are in the sagittal plane.

As in the plane surface problem, the equations to be considered are the elastic equation of motion, Maxwell's equations, and the constitutive equations. Wave forms are found satisfying these equations and are then coupled at the interface through the boundary conditions. Since we have been through this procedure, we will attempt a parallel development of the exact electrodynamic analysis and the electrostatic analysis.

In cylindrical coordinates the elastic equation of motion [8] is

$$- \rho\omega^2 u = \frac{\partial T_{rz}}{\partial r} + \frac{1}{r}\frac{\partial T_{\theta z}}{\partial \theta} + \frac{\partial T_{rz}}{\partial r} \qquad (42)$$

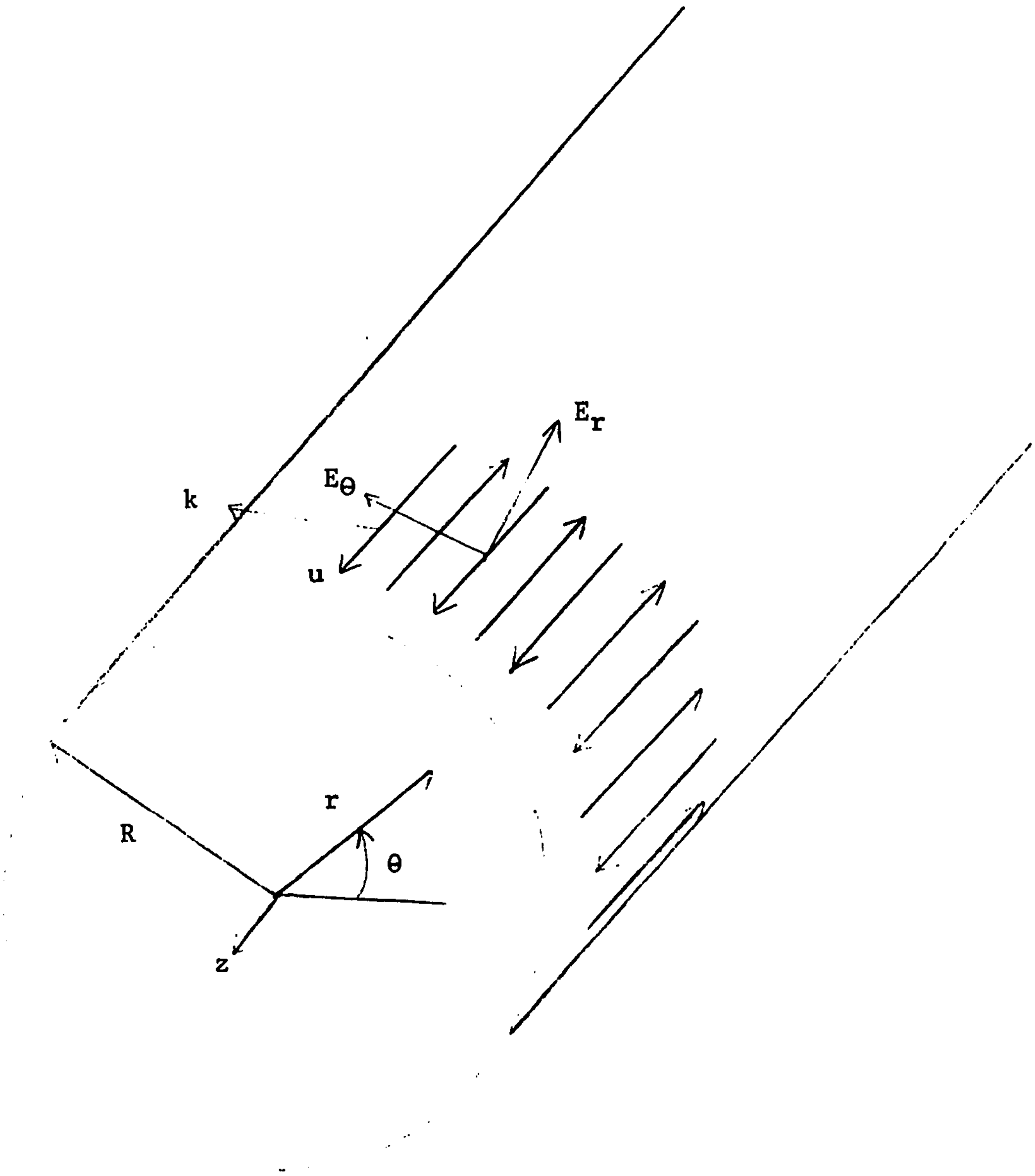

Figure 2. Cylindrical surface wave geometry

The wave is uniform in the axial direction so there is no z derivative. The constitutive relations [8] are

$$T_{rz} = c \frac{\partial u}{\partial r} - eE_r \tag{43a}$$

$$T_{\theta z} = \frac{c}{r} \frac{\partial u}{\partial \theta} - eE_\theta \tag{43b}$$

$$D_r = e \frac{\partial u}{\partial r} + \epsilon E_r \tag{43c}$$

$$D_\theta = \frac{e}{r} \frac{\partial u}{\partial \theta} + \epsilon E_\theta \quad . \tag{43d}$$

Substituting Eqs. 43c and 43d into the Maxwell equation $\bar{\nabla} \cdot \bar{D} = 0$ leads to

$$e\nabla^2 u + \epsilon \bar{\nabla} \cdot \bar{E} = 0 \ . \tag{44}$$

To satisfy this, we separate the electric field into an irrotational field and a solenoidal field

$$\bar{E} = \bar{E}^i + \bar{E}^s \tag{45a}$$

where

$$\bar{\nabla} \cdot \bar{E}^s = 0 \tag{45b}$$

and

$$\bar{\nabla} \times \bar{E}^i = \bar{\nabla} \times (- \bar{\nabla}\phi) = 0 \ . \tag{45c}$$

$\bar{E}^s$ is, at this point, an arbitrary field which satisfies Eq. 45b. If the electrostatic potential is proportional to the elastic displacement,

$$\phi = \frac{e}{\epsilon} u \tag{46}$$

Eq. 44 is identically zero.

With these potentials the constitutive relations 43 become

$$T_{rz} = \bar{c}\,\frac{\partial u}{\partial r} - eE_r^s \qquad (47a)$$

$$T_{\theta z} = \frac{\bar{c}}{r}\,\frac{\partial u}{\partial r} - eE_\theta^s \qquad (47b)$$

$$D_r = \epsilon E_r^s \qquad (47c)$$

$$D_\theta = \epsilon E_\theta^s\ . \qquad (47d)$$

where $\bar{c} = c + \frac{e^2}{\epsilon}$ is the piezoelectric stiffened elastic constant.

Substituting the stress constitutive relations 47a and 47b into the equation of motion, Eq. 42, leads to an equation involving u only,

$$\nabla^2 u + k_a^2 u = 0 \qquad (48)$$

where ∇^2 is the cylindrical two-dimensional Laplacian operator and $k_a = \sqrt{\frac{\rho}{\bar{c}}}$ as in the previous sections.

Letting $u = U(r)V(t)$ in Eq. 48 and using the method of separation of variables leads to two equations, a harmonic equation in θ and the Bessel equation

$$\frac{d^2 Z_n(k_a r)}{dr^2} + \frac{1}{r}\,\frac{dZ_n(k_a r)}{dr} + (k_a^2 - \frac{n^2}{r^2})\ Z_n(k_a r) = 0 \qquad (49)$$

where the kind of Bessel function, $Z_n(k_a r)$, is to be determined later. Thus, the elastic wave propagating around a cylinder of radius R will have to form

$$u = Z_n(k_a r)e^{jn\theta} \qquad (50)$$

where n = kR is the angular propagation constant. The relation between n and the phase velocity is

$$v = \frac{\omega}{k} = \frac{\omega R}{n} .$$

The angular propagation constant n is not limited to integers if we interpret the multivalued function

$$e^{jn\theta} = e^{jn(\theta' + p2\pi)}$$

where p is an integer and $0 \leq \theta' < 2\pi$, as a wave making its p^{th} revolution around the cylinder.

Accompanying the elastic wave is the irrotational field

$$\overline{E}^{i} = - \frac{e}{\epsilon} \overline{\nabla} u \quad . \tag{51}$$

The solenoidal field and the elastic displacement are not coupled in the bulk medium but, as will be shown later, they are coupled at the surface of the medium.

Up to this point the analysis is valid for both the electrodynamic Maxwell's equations and the electrostatic approximation. In the exact solution Maxwell's equations are

$$\overline{\nabla} \times \overline{E}^{s} = j \, \omega \, \mu_{o} \, \overline{H} \tag{52a}$$

$$\overline{\nabla} \times \overline{H} = - j \, \omega \, \overline{D} \tag{52b}$$

Eliminating the magnetic field intensity and substituting the flux density constitutive relations 47c and 47d into the results leads to an equation involving $\overline{E}^{s}$ only,

$$\nabla^2\overline{E}^s + k_b^2\overline{E}^s = 0 \tag{53}$$

where $k_b = \omega\sqrt{\mu_o\epsilon}$. But, since the cylindrical vector components of $\overline{E}^s$ do not satisfy the scalar Helmholtz equation, we let

$$\overline{E}^s = -\overline{\nabla} \times \overline{z}\,\psi \tag{54}$$

where $\overline{z}$ is a unit vector in the axial direction and ψ is a solution of the Helmholtz equation

$$\nabla^2\psi + k_b^2\,\psi = 0\ . \tag{55}$$

Since Eq. 55 is similar to Eq. 48 the potential will have the form

$$\psi = Z_n(k_b r)e^{jn\theta} \tag{56}$$

where $Z_n(k_b r)$ is again a Bessel function of order $n = kR$.

When we use the electrostatic approximation, Eq. 52a becomes

$$\overline{\nabla} \times \overline{E}^s = 0\ . \tag{57}$$

This is identically zero if we let

$$\overline{E}^s = -\overline{\nabla}\,\psi\ . \tag{58}$$

Since $\overline{\nabla} \cdot \overline{E}^s = 0$, ψ is a solution of Laplace's equation

$$\nabla^2\,\psi = 0 \tag{59}$$

The form of ψ having the same θ dependence as the elastic wave is

$$\psi = r^{\pm n}\,e^{jn\theta} \tag{60}$$

The external electric fields are determined in a similar manner. The electrodynamic Maxwell's equation lead to

$$\hat{\bar{E}} = - \bar{\nabla} \times \bar{z} \, \hat{\psi} \tag{61a}$$

where

$$\hat{\psi} = Z_n(k_c r) e^{jn\theta} \, . \tag{61b}$$

The electrostatic approximation gives

$$\hat{\bar{E}} = - \bar{\nabla} \, \hat{\psi} \tag{62a}$$

where

$$\hat{\psi} = r^{\pm n} \, e^{jn\theta} \, . \tag{62b}$$

The choice of the sign on the exponent will be determined by whether the potential must be finite at the origin or finite at infinity.

So far the kind of Bessel function has not been chosen. This choice will be determined by the behavior of the function as the argument approaches zero or infinity. For physically realizable solutions the functions must remain finite. The Bessel function of the first kind is the only one finite at the origin. For real arguments all Bessel functions are zero at infinity. However, if the medium is elastically or electrically lossy, the argument is complex and only the Bessel function of the third kind, Hankel's function, is zero at infinity. Because of the previous choice of $e^{-j\omega t}$ time dependence, Hankel's functions of the first kind represents outward propagating waves. Therefore, in the electrodynamic case, the waveforms for propagation around a solid cylinder are

$$u = AJ_n(k_a r)e^{jn\theta} \quad , \quad r \le R \tag{63a}$$

$$\psi = BJ_n(k_b r)e^{jn\theta} \quad , \quad r \le R \tag{63b}$$

$$\hat{\psi} = CH_n^{(1)}(k_c r)e^{jn\theta} \quad , \quad r > R \tag{63c}$$

The superscript denoting the Hankel function of the first kind is dropped henceforth.

The electrostatic wave forms are

$$u = AJ_n(k_a r)e^{jn\theta} \quad , \quad r \le R \tag{64a}$$

$$\psi = Br^n e^{jn\theta} \quad , \quad r \le R \tag{64b}$$

$$\hat{\psi} = Cr^{-n} e^{jn\theta} \quad , \quad r > R \tag{64c}$$

To find the wave amplitudes B and C in terms of A and the propagation constant n in terms of k_a, k_b, and k_c, we satisfy the boundary conditions at $r = R$. These conditions are vanishing stress,

$$T_{rz} = \bar{c}\,\frac{\partial u}{\partial r} - eE_r^s \qquad \text{at } r = R \tag{65a}$$

continuous tangential electric field

$$E_\theta^i + E_\theta^s = \hat{E}_\theta \qquad \text{at } r = R \tag{65b}$$

and continuous normal component of the electric flux density

$$\epsilon E_r^i = \epsilon_o \hat{E}_r \qquad \text{at } r = R\ . \tag{65c}$$

The subscripts denote the vector component and the superscripts indicate whether the field is given by Eq. 51 or Eq. 54 and 58. The external field, denoted by a circumflex, is given by Eq. 61 and 62.

Substituting wave forms 63 into the boundary conditions 65 leads to the following amplitude ratios and secular propagation constant equation for the electrodynamic case:

$$\frac{B}{A} = \frac{j\bar{c}x_a J_n'(x_a)}{e\, n\, J_n(x_b)} \tag{66a}$$

$$\frac{C}{A} = \frac{j\epsilon\bar{c}x_a J_n'(x_a)}{\epsilon_o e\, n\, H_n(x_c)} \tag{66b}$$

$$n^2K^2 = \frac{x_a J_n'(x_a)}{J_n(x_a)} \left[\frac{x_b J_n'(x_b)}{J_n(x_b)} - \frac{\epsilon}{\epsilon_o} \frac{x_c H_n'(x_c)}{H_n(x_c)} \right] \tag{66c}$$

where

$$x_a = k_a R, \; x_b = k_b R, \text{ and } x_c = k_c R \; .$$

For the electrostatic case, substituting wave forms 64 into the boundary conditions 65 gives

$$\frac{B}{A} = - \frac{\bar{c}x_a J_n'(x_a)}{e\; n\; R^n} \tag{67a}$$

$$\frac{C}{A} = \frac{\epsilon\; \bar{c}\; R^n x_a J_n'(x_a)}{\epsilon_o\; e\; n} \tag{67b}$$

$$nK^2 = \left(1 + \frac{\epsilon}{\epsilon_o}\right) \frac{x_a J_n'(x_a)}{J_n(x_a)} \tag{67c}$$

A closer examination of the propagation constant equation gives the following information. If the piezoelectric coupling constant is zero, Eq. 66c is separated into two equations,

$$J_n'(x_a) = 0 \tag{68a}$$

$$\frac{x_b J_n'(x_b)}{J_n(x_b)} - \frac{\epsilon}{\epsilon_o} \frac{x_c H_n'(x_c)}{H_n(x_c)} = 0 \,. \tag{68b}$$

Eq. 68a gives the modes of a purely elastic wave propagating around a solid cylinder. These waves are similar to the "whispering gallery" modes, where sound waves are continuously reflected by the surface as they propagate around the inside of the cylinder. For a given radius, a finite number of positive real propagation constants satisfy this equation. The phase velocities of these modes are faster than the velocity of the uniform plane wave since n is less than x_a [9]. Eq. 68b gives the radiating modes of an electromagnetic wave propagating around a dielectric cylinder.

If the piezoelectric coupling constant is not zero, the elastic and electromagnetic modes are perturbed by the coupling at the surface. In addition, there is a new mode which for a large enough radius has a phase velocity slower than the velocity of a volume wave. This is the Bleustein-Galyaev wave. It will be shown later that as the radius decreases, the phase velocity of the surface wave increases.

Since the electrodynamic secular equation contains the complex Hankel function, the propagation constant is complex. This means that as the wave propagates around the cylinder its amplitude decays exponentially as energy is radiated away from the cylinder by the external electromagnetic field. The electrostatic approximation does not show this since the external field is irrotational and can not be radiative.

The electrostatic approximation is a limit of the electrodynamic solution as the velocity of the solenoidal field goes to infinity (propagation constant k_b and k_c go to zero). It is easily shown that if the small argument approximations for $J_n(k_b r)$, $H_n(k_c r)$, and their derivatives are substituted in the electrodynamic solutions, the electrostatic solutions result.

2. Conducting interface

The solutions for a very thin conductor on the surface of the cylinder may be found by substituting wave forms 63 and 64 into the boundary conditions

$$T_{rz} = \overline{c}\,\frac{\partial u}{\partial r} - eE_r^s = 0 \qquad \text{at} \quad r = R \tag{69a}$$

$$E_\theta^i + E_\theta^s = 0 \qquad \text{at} \quad r = R \;. \tag{69b}$$

Alternatively we can derive them directly from solutions 66 and 67 by letting the permittivity of the external free space approach infinity. For the electrodynamic case, the wave amplitude ratio and secular equation are

$$\frac{B}{A} = \frac{j\,\overline{c}\,x_a J_n'(x_a)}{e\,n\,J_n(x_b)} \tag{70a}$$

$$n^2K^2 = \frac{x_a J_n'(x_a)}{J_n(x_a)}\;\frac{x_b J_n'(x_b)}{J_n(x_b)} \;. \tag{70b}$$

With no external electric field, there can be no radiation away from the cylinder and the propagation constant is real.

The electrostatic solutions for a conducting interface are

$$\frac{B}{A} = -\frac{\bar{c}\, x_a J_n'(x_a)}{e\, n\, R^n} \tag{71a}$$

$$nK^2 = \frac{x_a J_n'(x_a)}{J_n(x_a)} \quad . \tag{71b}$$

B. Propagation Around a Cylindrical Cavity

The analysis of a surface wave propagating around a cylindrical cavity is similar to the analysis of the solid cylinder surface wave. Figure 2 shows the configuration of this problem. Free space occupies the cavity $r < R$ and the piezoelectric medium extends from $r = R$ to infinity.

The elastic and electromagnetic waveforms are chosen to insure finite values at the origin and at infinity. The waveforms are then substituted into boundary conditions 65 or 69. The free space interface results are listed below.

Electrodynamic solutions:

$$u = AH_n(k_a r)e^{jn\theta} \quad , \quad r \geq R \tag{72a}$$

$$\psi = BH_n(k_b r)e^{jn\theta} \quad , \quad r \geq R \tag{72b}$$

$$\hat{\psi} = CJ_n(k_c r)e^{jn\theta} \quad , \quad r < R \tag{72c}$$

$$\frac{B}{A} = \frac{j\,\bar{c}\,x_a H_n'(x_a)}{e\,n\,H_n(x_b)} \tag{72d}$$

$$\frac{C}{A} = \frac{j\,\epsilon\,\bar{c}\,x_a H_n'(x_a)}{e\,n\,H_n(x_b)} \tag{72e}$$

$$n^2 K^2 = \frac{x_a H_n'(x_a)}{H_n(x_a)} \left[\frac{x_b H_n'(x_b)}{H_n(x_b)} - \frac{\epsilon}{\epsilon_o} \frac{x_c J_n'(x_c)}{J_n(x_c)} \right] . \tag{72f}$$

Electrostatic solutions:

$$u = A H_n(k_a r) e^{jn\theta} \quad , \quad r \geq R \tag{73a}$$

$$\psi = B r^{-n} e^{jn\theta} \quad , \quad r \geq R \tag{73b}$$

$$\hat{\psi} = C r^{n} e^{jn\theta} \quad , \quad r < R \tag{73c}$$

$$\frac{B}{A} = \frac{\bar{c}\,R^n x_a H_n'(x_a)}{e\,n} \tag{73d}$$

$$\frac{C}{A} = - \frac{\epsilon\,\bar{c}\,x_a H_n'(x_a)}{\epsilon_o\,e\,n\,R^n} \tag{73e}$$

$$nK^2 = - \left(1 + \frac{\epsilon}{\epsilon_o}\right) \frac{x_a H_n'(x_a)}{H_n(x_a)} . \tag{73f}$$

The surface wave propagation constant is given by the largest value of n that satisfies the secular equations 72f or 73f. Eq. 73f shows that, even in the electrostatic case, the propagation constant is complex. The amplitude of the surface wave will decay exponentially in the θ direction as energy is radiated away from the surface into the piezoelectric medium by the elastic wave. This is similar to the behavior of a purely elastic Rayleigh wave[1] propagating around a cylindrical cavity [10,11]. Surface waves of this type have been called "leaky" surface waves [11]. In the exact electrodynamic case, both the elastic wave and the electric wave radiate energy away from the surface into the piezoelectric medium.

The propagation constant equations for a conducting interface are given by letting ε_o go to infinity in Eqs. 72f and 72e. The results are

$$n^2K^2 = \frac{x_a H_n'(x_a)}{H_n(x_a)} \frac{x_b H_n'(x_b)}{H_n(x_b)} \tag{74}$$

from the electrodynamic equations, and

$$nK^2 = -\frac{x_a H_n'(x_a)}{H_n(x_a)} \tag{75}$$

from the electrostatic approximation.

If the piezoelectric coupling constant is zero, Eq. 74 reduces to two equations,

$$H_n'(x_a) = 0 \tag{76a}$$

$$H_n'(x_b) = 0 \,. \tag{76b}$$

The roots of Eq. 76a give the propagation constants for purely elastic waves that are diffracted around the cylindrical cavity. Waves of this type have been called "creeping waves" [12,13]. Similarly Eq. 76b

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[1] The elastic displacement of a Rayleigh wave is parallel to the sagittal plane.

represents electromagnetic waves in a dielectric that are diffracted around a conducting cylinder. The propagation constants given by Eqs. 76 are complex.

C. Large Radii Approximations

The propagation constant equations involve ratios of Bessel functions and in some cases, Bessel functions whose order is complex. Therefore, an approximation is desired that gives the propagation constant explicitly, especially for radii much larger than the elastic wavelength where the order and argument of the Bessel function are large. Such approximations are derived here by using asymptotic expansions of Bessel functions of large order and argument and keeping only the terms up to order $\frac{1}{R}$. The derivations for the following terms that occur in the propagation constant equations are given in the Appendix:

$$\frac{xJ_n'(x)}{J_n(x)} \approx [1 + \frac{x^2}{2(n^2-x^2)^{3/2}}](n^2 - x^2)^{\frac{1}{2}} \tag{77a}$$

$$\frac{xH_n'(x)}{H_n(x)} \approx -[1 - \frac{x^2}{2(n^2-x^2)^{3/2}} - j\, e^{2f(n,x)}](n^2 - x^2)^{\frac{1}{2}} \tag{77b}$$

where

$$f(n,x) = (n^2 - x^2)^{\frac{1}{2}} - n \cosh^{-1} \frac{n}{x} \; . \tag{77c}$$

Substituting these expressions into the electrodynamic equations 66c and 72f, squaring the equations, and solving for the propagation constant keeping only the terms up to order $\frac{1}{R}$ leads to the desired expressions.

(the Appendix gives the details of these derivations). For propagation around a free interface solid cylinder,

$$k \approx k_s(1 - \delta) + jk' \tag{78a}$$

where

$$\delta = \frac{1}{2(k_s^2 - k_a^2)^{\frac{1}{2}} R} \tag{78b}$$

$$k' = \frac{(k_s^2 - k_a^2)k_s}{(1 + \frac{\epsilon_o}{\epsilon})k_a^2} e^{2f(k_s, k_c)R} \tag{78c}$$

and k_s, the propagation constant for a surface wave on a flat surface, is given by Eq. 39c,

$$k_s = \frac{k_a}{[1 - \frac{K^4}{(1 + \frac{\epsilon}{\epsilon_o})^2}]^{\frac{1}{2}}} . \tag{78d}$$

Since δ is positive, the curvature of the surface decreases k (increases the phase velocity). The imaginary part of k comes from radiation of energy away from the surface by an external electromagnetic field. It can be shown that $f(k_s, k_c)$ is negative. Therefore, as the radius increases, δ goes to zero as $\frac{1}{R}$, k' goes to zero exponentially, and the propagation constant approaches the propagation constant of a surface wave on a flat surface. Because of the respective dependence of k' and δ on R, $k' \ll \delta$ and the electrostatic approximation with $k' = 0$ is valid.

When the interface is covered with a conductor we have

$$k \approx k_s(1 - \delta) \tag{79a}$$

where from Eq. 41b

$$k_s = \frac{k_a}{(1 - K^4)^{\frac{1}{2}}} \quad . \tag{79b}$$

For propagation around a cylindrical cavity we use Eqs. 72f and 77 to derive

$$k \approx k_s(1 + \delta) + jk' + jk'' \tag{80a}$$

where

$$k' = \frac{(k_s^2 - k_a^2)k_s}{(1 + \frac{\varepsilon}{\varepsilon_o})k_a^2} e^{2f(k_s,k_b)R} \tag{80b}$$

$$k'' = \frac{(k_s^2 - k_a^2)k_s}{k_a^2} e^{2f(k_s,k_a)R} \tag{80c}$$

and δ and k_s are given by Eqs. 78b and 78d. Here we see that the curvature increases k and decreases the phase velocity.

For propagation around a cylindrical cavity with a metallized surface, the propagation constant has the same form as Eq. 80a with

$$k' = \frac{(k_s^2 - k_a^2)k_s}{k_a^2} e^{2f(k_s,k_b)R}$$

and k_s and k' given by Eq. 79b and 80c, respectively.

Eq. 80a has two imaginary terms, one from radiation of electromagnetic energy away from the surface and the other from radiation of mechanical (elastic) energy. The relative magnitudes of the two forms of radiation can be compared by examining the ratios of the exponentials in Eqs. 80b and 80c:

$$\frac{f(k_s,k_b)}{f(k_s,k_a)} = \frac{(k_s^2 - k_b^2)^{\frac{1}{2}} - k_s \cosh^{-1}\frac{k_s}{k_b}}{(k_s^2 - k_a^2)^{\frac{1}{2}} - k_s \cosh^{-1}\frac{k_s}{k_a}}$$

$$\approx \frac{- k_s \cosh^{-1}\frac{k_s}{k_b}}{(k_s^2 - k_a^2)^{\frac{1}{2}} - k_s \cosh^{-1}\frac{k_s}{k_a}} \gg 1 \tag{81}$$

since $k_s \approx k_a \gg k_b \approx k_c$. Therefore, $k' \ll k''$ and most of the energy is radiated in the form of an elastic wave. So for large radii the electrostatic approximation with $k' = 0$ is valid.

In a purely elastic medium the Bleustein-Galyaev wave does not exist, but the Rayleigh surface wave does exist. It has been shown that the Rayleigh wave propagation constant has radius dependence similar to that shown here for the electrostatic surface wave [10].

Figure 3 presents the propagation constant calculated from large radius approximation, Eqs. 79a, and also from the electrostatic propagation constant equation, Eq. 71b, for the poled ceramic PZT-5A with a conducting interface. For large radii the curves coincide. Note that for small radii, the surface wave propagation constant becomes less than the uniform plane wave propagation constant.

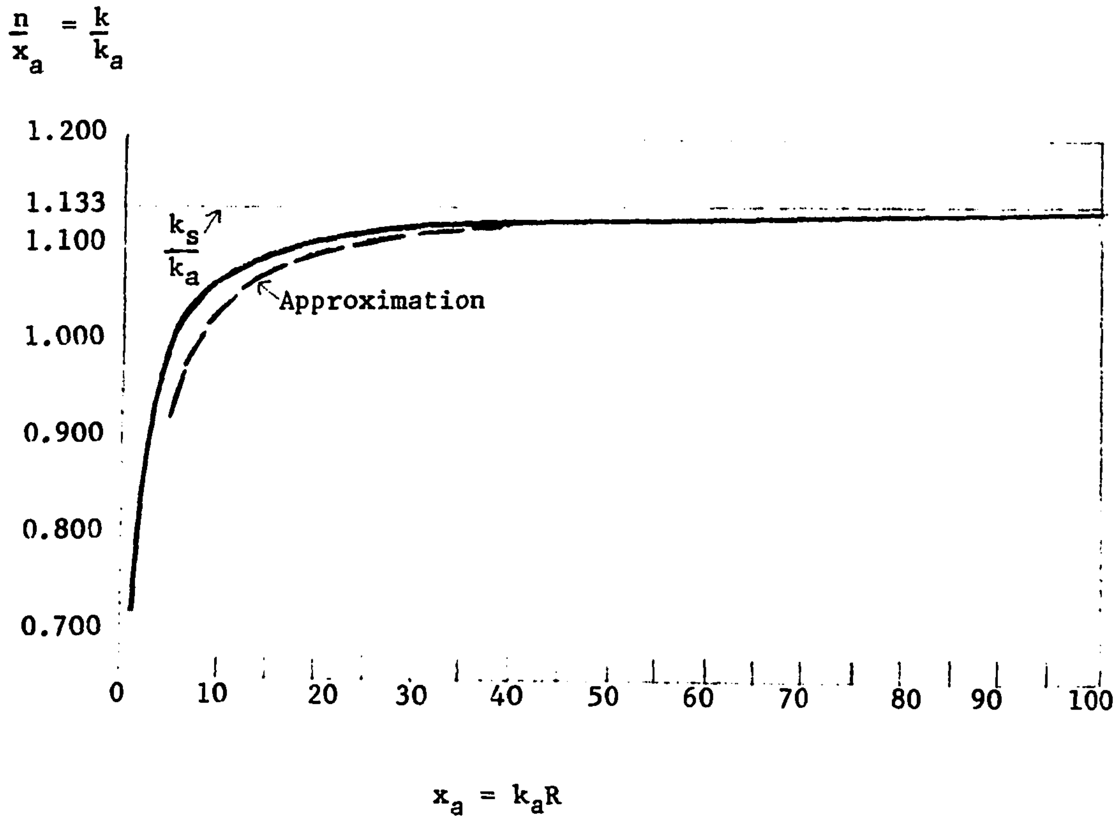


Figure 3. Electrostatic and large radius approximation propagation constant for PZT-5A with a conducting interface

IV. EXCITATION OF BLEUSTEIN-GULYAEV WAVES ON A FLAT SURFACE

A. Integral Solutions

The surface wave solutions derived in the previous sections actually pertain only to cases where sources or scatterers are remote. They can, however, be combined with other wave functions to treat the general problem of excitation. This section is devoted to this treatment.

Through the coupling of the fields at the surface of the piezoelectric medium, both electromagnetic and elastic sources will excite surface waves. Figure 4 shows three sources, an electromagnetic and elastic source in the piezoelectric at $x_2 = h$ and an electromagnetic source at $x_2 = -h$. These are line sources extending to infinity parallel to the x_3 axis (not shown).

The fields from an arbitrary source can be expressed as a sum of plane waves which satisfy the elastic and electromagnetic equations of motion. To include the propagation of electromagnetic waves, the exact electrodynamic wave equations are used. The source wave forms are

$$u^o = \int_{-\infty}^{\infty} u_o(k_1) e^{\pm \nu_a (x_2 - h) + jk_1 x_1} \, dk_1 \quad , \quad x_2 \lessgtr h \tag{82a}$$

$$\psi^o = \int_{-\infty}^{\infty} \psi_o(k_1) e^{\pm \nu_b (x_2 - h) + jk_1 x_1} \, dk_1 \quad , \quad x_2 \lessgtr h \tag{82b}$$

$$\hat{\psi}^o = \int_{-\infty}^{\infty} \hat{\psi}_o(k_1) e^{\mp \nu_c (x_2 + h) + jk_1 x_1} \, dk_1 \quad , \quad x_2 \gtrless h \tag{82c}$$

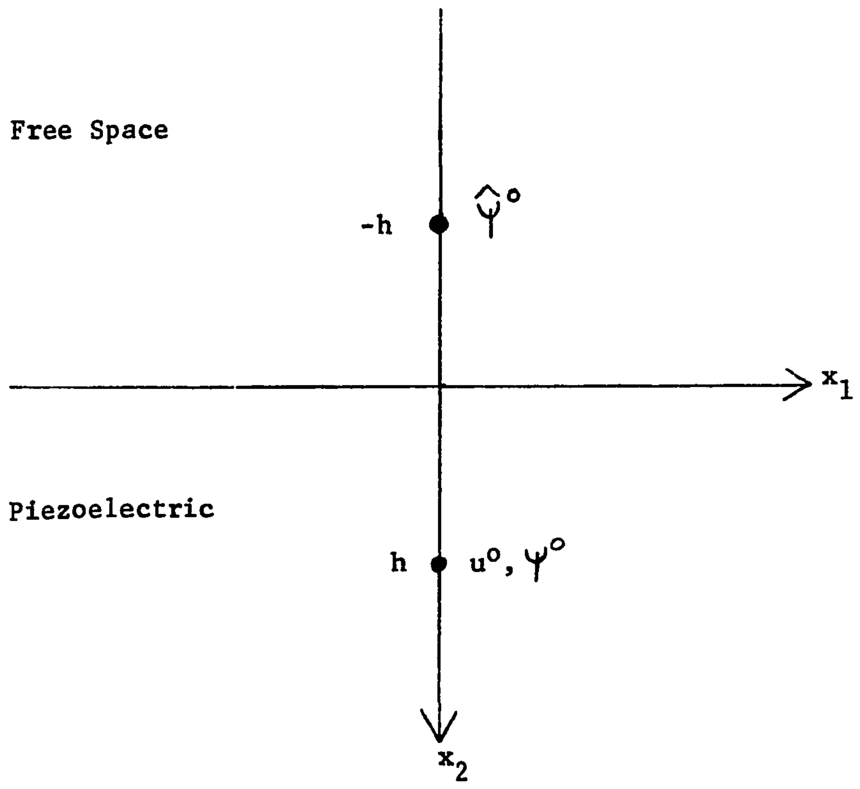


Figure 4. Elastic and electromagnetic line sources

where

$$\nu_a = \sqrt{k_1^2 - k_a^2} = -j\sqrt{k_a^2 - k_1^2}$$

$$\nu_b = \sqrt{k_1^2 - k_b^2} = -j\sqrt{k_b^2 - k_1^2}$$

$$\nu_c = \sqrt{k_1^2 - k_c^2} = -j\sqrt{k_c^2 - k_1^2} .$$

The signs on the radicals are chosen to insure a wave radiating away from the sources and to insure that the integrals converge. Note that the sources are constructed from uniform plane waves when $k_1 < k_a$, for example, and inhomogeneous waves when $k_1 > k_a$. As a simple illustration of how Eqs. 82 might be applied, consider the consequences of cylindrically symmetrical elastic excitation. A disturbance from such a source would propagate away from such a source in such a manner that surfaces of constant phase would be cylindrical and concentric with the source. Consequently, the field u^o would be described by a Hankel function of the first kind. The integral will, in fact, become equal to such a function if $u_o(k_1)$ is chosen to proportional to $\frac{1}{\nu_a}$ [14].

When an electromagnetic source wave impinges on the piezoelectric-free space interface, part of the electromagnetic energy in the wave will be reflected from the interface, part will be transmitted across the interface, and part of the electromagnetic energy will be converted to mechanical energy through the interaction at the interface and transmitted or reflected in the form of an elastic wave. Likewise, an elastic source wave impinging on the interface will produce a reflected elastic wave and a reflected and transmitted electromagnetic wave. The reflected and transmitted fields are also expressed as a sum of plane waves:

$$u = \int_{-\infty}^{\infty} A(k_1) e^{-\nu_a x_2 + jk_1 x_1} dk_1 \tag{83a}$$

$$\psi = \int_{-\infty}^{\infty} B(k_1) e^{-\nu_b x_2 + jk_1 x_1} dk_1 \tag{83b}$$

$$\hat{\psi} = \int_{-\infty}^{\infty} C(k_1) e^{\nu_c x_2 + jk_1 x_1} dk_1 \tag{83c}$$

Waveforms 82 and 83 satisfy the wave equations since their integrands satisfy the wave equations. To find the amplitudes A, B, and C in terms of the sources, we substitute the waveforms into the three boundary conditions given by Eqs. 18. When this is done, we obtain three integrals which must vanish for all values of x_1. In general, this will only occur if the integrands vanish. In other words, every uniform and inhomogeneous plane wave of the continuous spectrum that constitutes the source field must satisfy the boundary conditions. This leads to the following equations written in matrix form:

$$\begin{bmatrix} \nu_a \ , & \frac{je}{c} k_1 \\ \frac{je}{\epsilon} k_1 \ , & -(\nu_b + \frac{\epsilon}{\epsilon_o} \nu_c) \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} =$$

$$\begin{bmatrix} \nu_a \ , & -\frac{je}{c} k_1 \ , & 0 \\ -\frac{je}{\epsilon} k_1 \ , & -(\nu_b - \frac{\epsilon}{\epsilon_o} \nu_c) \ , & -2\nu_c \end{bmatrix} \begin{bmatrix} u_o(k_1) e^{-\nu_a h} \\ \psi_o(k_1) e^{-\nu_b h} \\ \hat{\psi}(k_1) e^{\nu_c h} \end{bmatrix} \tag{84a}$$

From the continuity of the normal electric flux density the remaining amplitude is

$$C = \frac{\varepsilon}{\varepsilon_o}\left[B + \psi_o(k_1)e^{-\nu_b h}\right] - \hat{\psi}_o e^{-\nu_c h} \tag{84b}$$

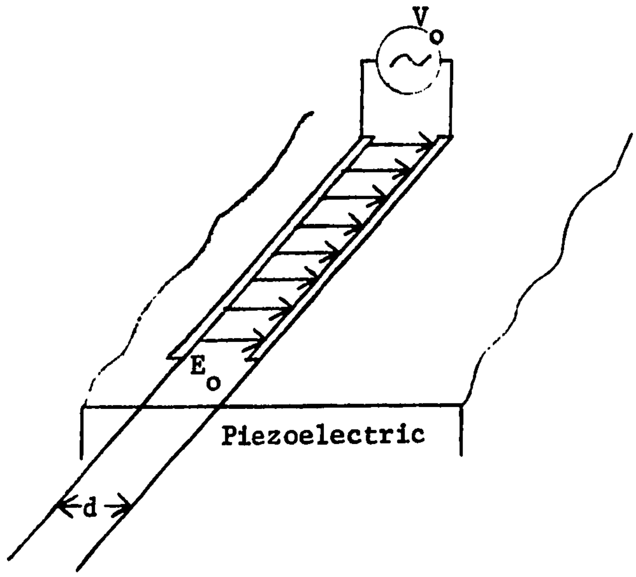
Using Cramer's rule to solve for the amplitudes A and B in terms of the sources and substituting the results into waveforms 83 gives the integral solutions to this problem. The solutions for internal electromagnetic or elastic sources and a metallized surface are found by letting $\frac{\varepsilon}{\varepsilon_o}$ be zero in Eqs. 84. Evaluation of the integrals will be demonstrated in the following sections.

B. Electric Dipole Source

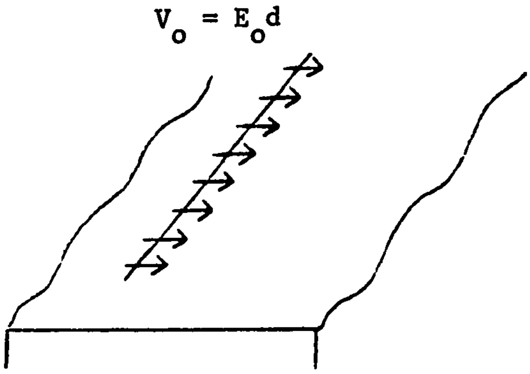
1. Integral solution

In this section we will calculate the waves excited by a line of electric dipoles at $x_2 = - h$. There is a practical reason for choosing this source as an example. Most surface wave devices employ interdigital electrodes to excite the surface wave. These consist of thin, conducting strips deposited on or near the surface as shown for one pair of electrodes in Figure 5a. The field between the electrodes is $E_o = \frac{V_o}{d}$. Letting d approach zero while keeping the voltage constant results in the dipole line source shown in Figure 5b. The effect of the conductors giving rise to the field is disregarded. The vector potential of a line of electric dipoles at $x_2 = - h$ is given by Eq. 82c with $\hat{\psi}_o(k_1) = \hat{\psi}_o$ where

$$\hat{\psi}_o = - \frac{E_o d}{4\pi} \quad . \tag{85}$$



(a) Electrode pair



(b) Dipole approximation

Figure 5. External electromagnetic source

The source, reflected and transmitted fields from Eqs. 82c, 83, and 84 are then

$$\hat{\psi}^{o} = \hat{\psi}_{o} \int_{-\infty}^{\infty} e^{-\nu_c(x_2 + h) + jk_1x_1}\, dk_1 \tag{86a}$$

$$u = \frac{j2e}{\bar{c}} \hat{\psi}_{o} \int_{-\infty}^{\infty} \frac{\nu_c k_1}{D(k_1)}\, e^{-\nu_a x_2 - \nu_c h + jk_1x_1}\, dk_1 \tag{86b}$$

$$\psi = -2\hat{\psi}_{o} \int_{-\infty}^{\infty} \frac{\nu_a \nu_c}{D(k_1)}\, e^{-\nu_b x_2 - \nu_c h + jk_1x_1}\, dk_1 \tag{86c}$$

$$\hat{\psi} = -\hat{\psi}_{o} \int_{-\infty}^{\infty} \left[\frac{2\,\epsilon\,\nu_a\nu_c}{\epsilon_o\, D(k_1)} + 1\right] e^{\nu_c(x_2 - h) + jk_1x_1}\, dk_1 \tag{86d}$$

where

$$D(k_1) = K^2k_1^2 - \nu_a(\nu_b + \frac{\epsilon}{\epsilon_o}\nu_c) \tag{86e}$$

Note that setting Eq. 86e equal to zero gives propagation constant equation, Eq. 19c, for a surface wave.

2. Surface wave

Evaluation of integrals 86 involves contour integration on the complex k_1 plane shown in Figure 6 where $k_1 = k_1' + jk_1''$, k_1' and k_1'' real. There are poles, labelled $k_1 = \pm k_s$, at the values of k_1 which satisfy $D(k_1) = 0$. The radicals ν_a, ν_b, and ν_c are multivalued functions so branch points at $k_1 = \pm k_a$, $\pm k_b$, and $\pm k_c$ are needed to insure that the integrands are analytic functions on the path of integration.

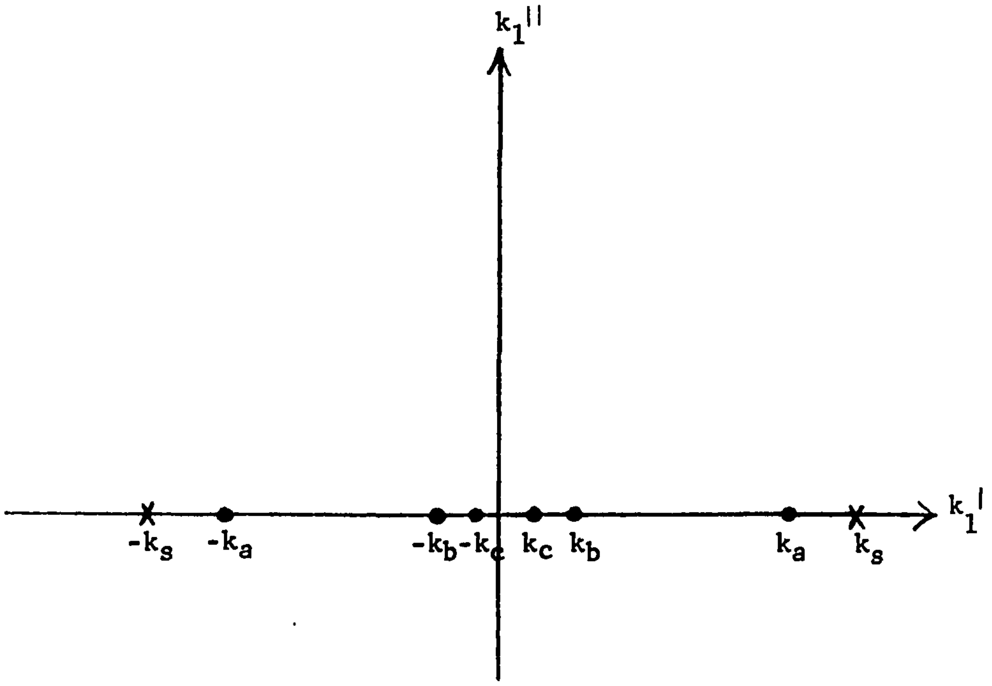


Figure 6. Complex k_1 plane

In order to use the residue theorem to evaluate the contribution for the pole we close the path of integration. This is more easily shown by letting k_a, k_b, k_c, and k_s have a small imaginary component which would be true if the medium were electrically and elastically lossy. After solutions are derived we can let the imaginary component be zero if that is desired. Because of the factor $e^{jk_1x_1} = e^{jk_1'x_1 - k_1''x_1}$ in the integrand, for waves traveling in the positive x_1 direction away from the source we must close the path of integration in the upper half plane as shown in Figure 7. Closing the path in the lower half plane will give similar solutions for waves traveling in the negative direction away from the source. Branch lines L_a, L_b, and L_c loop around the branch cuts. (The branch cuts are chosen here to extend from the branch points to infinity parallel to the positive imaginary axis.)

The residue theorem can now be written as

$$\oint f(k_1)dk_1 = 2\,\pi\,j\;\text{Res}\,[f(k_s)]$$

$$= \int_{-S}^{S} f(k_1)dk_1 + \int_{L_s} f(k_1)dk_1 + \int_{L_a} f(k_1)dk_1 \qquad (87)$$

$$+ \int_{L_b} f(k_1)dk_1 + \int_{L_c} f(k_1)dk_1$$

where $f(k_1)$ represents integrands of Eq. 86 and $\text{Res}[f(k_s)]$ denotes the residue of $f(k_1)$ at $k_1 = k_s$. The loop L_s is a semicircle of radius S. When we let $S \to \infty$ the integral over L_s goes to zero because of the factor $e^{-k_1''x_1}$. Therefore, the integral along the real axis becomes

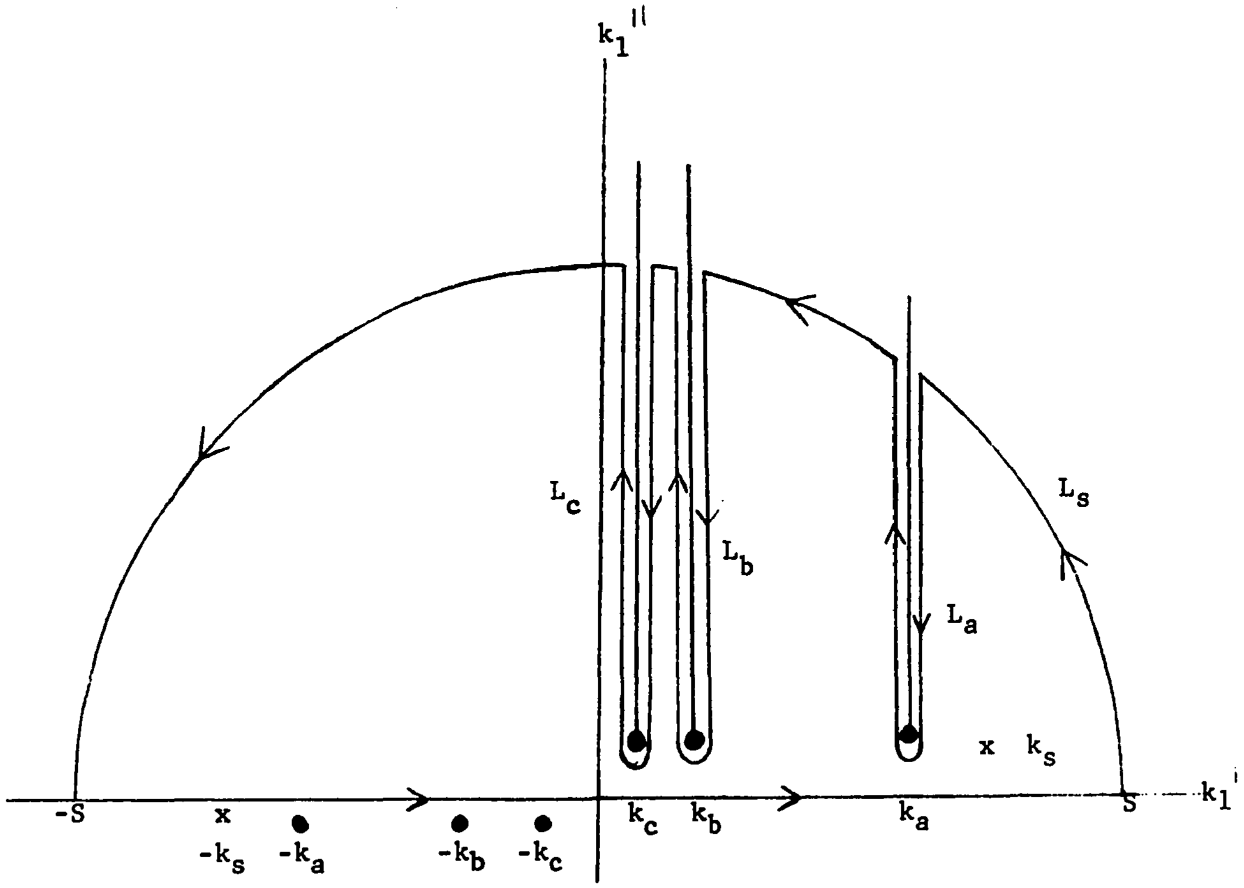


Figure 7. Path of integration

$$\int_{-\infty}^{\infty} f(k_1)dk_1 = 2\pi j \operatorname{Res}[f(k_s)] - \int_{L_a, L_b, L_c} f(k_1)dk_1 \tag{88}$$

The branch line integrals will be evaluated later. The residue at k_s of a function of the form

$$f(k_1) = \frac{N(k_1)}{D(k_1)} ,$$

where $N(k_1)$ and $D(k_1)$ are analytic functions of k_1, is

$$\operatorname{Res}[f(k_s)] = \frac{N(k_s)}{\left.\frac{d\,D(k_1)}{d\,k_1}\right|_{k_1 = k_s}} = \frac{N(k_s)}{D'(k_s)} \tag{89}$$

With Eq. 89 the integrals 86 become

$$u = -\frac{4\pi e \hat{\psi}_o \nu_c k_s}{\bar{c}\, D'(k_s)} e^{-\nu_c h - \nu_a x_2 + jk_s x_1} \tag{90a}$$

$$\psi = -\frac{4\pi j \hat{\psi}_o \nu_a \nu_c}{D'(k_s)} e^{-\nu_c h - \nu_b x_2 + jk_s x_1} \tag{90b}$$

$$\hat{\psi} = -\frac{4\pi j \epsilon \hat{\psi}_o \nu_a \nu_c}{\epsilon_o D'(k_s)} e^{-\nu_c h + \nu_c x_2 + jk_s x_1} \tag{90c}$$

where

$$D'(k_s) = 2K^2 k_s - \frac{k_s}{\nu_a}(\nu_b + \frac{\varepsilon}{\varepsilon_o}\nu_c) - \nu_a\left(\frac{k_s}{\nu_b} + \frac{\varepsilon}{\varepsilon_o}\frac{k_s}{\nu_c}\right)$$

and the radicals ν_a, ν_b, and ν_c are evaluated at $k_1 = k_s$. Eqs. 90 represent the surface wave excited by the electromagnetic source. The amplitude ratios of the electric potentials to the elastic displacement are the same as those derived for the free surface wave, Eqs. 19.

3. Lateral waves

To demonstrate the method used here to calculate branch line integrals, we will evaluate the elastic displacement at the surface with the source on the surface around the branch line L_a. With $x_2 = h = 0$, Eq. 86b is

$$u = \frac{j\,2\,e\,\hat{\psi}_o}{\bar{c}} \int_{L_a} \frac{\nu_c k_1 e^{jk_1x_1}}{K^2k_1^2 - \nu_a(\nu_b + \frac{\varepsilon}{\varepsilon_o}\nu_c)}\, dk_1 \quad . \tag{91}$$

On the path of integration, shown in Figure 7 let

$$k_1 = k_a + jk_a z \tag{92}$$

where k_a may now be real. When the path L_a is infinitesimally close to the branch cut, z is real and positive. Therefore, because of the $e^{-k_a x_1 z}$ term in the integrand, for large x_1 the main contribution comes from small values of z. (Since $k_a x_1 = \frac{2\pi}{\lambda_a} x_1$ where λ_a is the wave length of a uniform plane elastic wave, x_1 must be larger than a few wavelengths.) We can then use the following approximation for the radical:

$$\nu_a = \sqrt{(k_a + jk_a z)^2 - k_a^2} \approx k_a \sqrt{2j}\,(1 + \frac{jz}{4})(\pm\sqrt{z}) \tag{93}$$

To determine the sign of $\sqrt{z}$ on either side of the branch cut, let

$$z = |z|e^{j\theta} \tag{94}$$

where θ is measured from the branch cut as shown in Figure 8. Eq. 93 is then

$$\nu_a = k_a \sqrt{2j}\,(1 + \frac{j|z|e^{j\theta}}{4})\sqrt{|z|}\;e^{i\frac{\theta}{2}} \tag{95}$$

From this we see that at $\theta = 0$

$$\nu_a = k_a \sqrt{2j}\,(1 + \frac{jz}{4})\sqrt{z} = \bar{\nu}_a \tag{96a}$$

and at $\theta = 2\pi$

$$\nu_a = -\,k_a \sqrt{2j}\,(1 + \frac{jz}{4})\sqrt{z} = -\,\bar{\nu}_a \tag{96b}$$

Therefore, with $\nu_a = -\bar{\nu}_a$ of the right bank of the branch L_a cut, where $\bar{\nu}_a$ is given by Eq. 96a and with $\nu_a = \bar{\nu}_a$ on the left bank of the branch cut, Eq. 91 becomes

$$u = \frac{j\,2\,e\,\hat{\psi}_o}{\bar{c}} \left\{ \int_\infty^0 \left[\frac{\nu_c k_1 e^{jk_1 x_1}}{K^2 k_1^2 + \bar{\nu}_a(\nu_b + \frac{\varepsilon}{\varepsilon_o}\nu_c)} \right]_{k_1 = k_a + jk_a z} jk_a dz \right.$$

$$\left. + \int_0^\infty \left[\frac{\nu_c k_1 e^{jk_1 x_1}}{K^2 k_1^2 - \bar{\nu}_a(\nu_b + \frac{\varepsilon}{\varepsilon_o}\nu_c)} \right]_{k_1 = k_a + jk_a z} jk_a dz \right\} \tag{97}$$

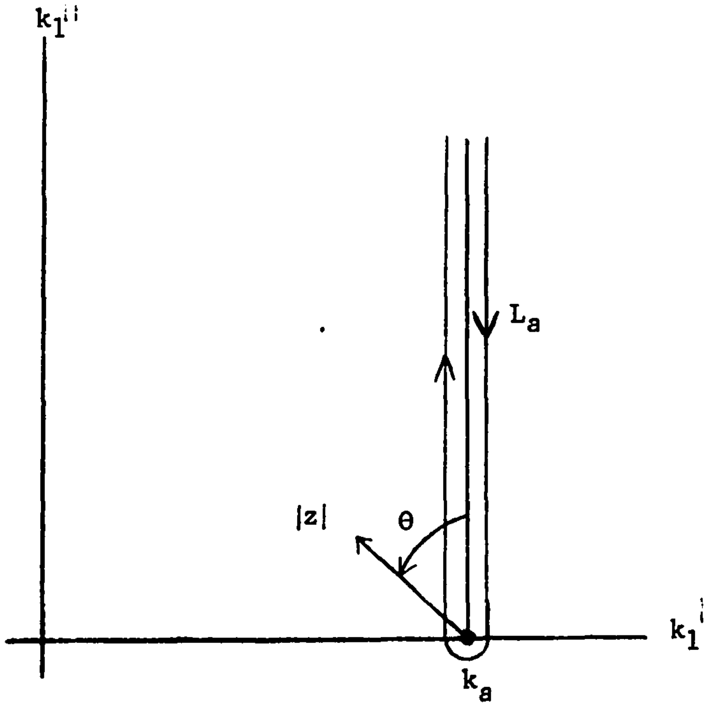


Figure 8. Geometry used in choosing the sign of $\sqrt{z}$

$$= -\frac{4 e \hat{\psi}_o}{\bar{c}} k_a^2 \sqrt{2j}\, e^{jk_a x_1} \int_0^\infty \sqrt{z}\, G(z) e^{-k_a x_1 z}\, dz$$

where

$$G(z) = \left[\frac{(1 + \frac{jz}{4})(\nu_b + \frac{\varepsilon}{\varepsilon_o}\nu_c)}{K^4 k_1^4 + \bar{\nu}_a^{-2}(\nu_b + \frac{\varepsilon}{\varepsilon_o}\nu_c)^2} \right]_{k_1 = k_a + jk_a z} \tag{98}$$

At first this may appear as complicated an integral as that in Eq. 91, but since for large x_1 the main contribution of the integral comes from small z we may expand G(z) in a Taylor series around z = 0,

$$G(z) = G(0) + G'(0)z + \frac{G''(0)}{2!} z^2 + \cdots . \tag{99}$$

If we apply the integral definition of the gamma function

$$\frac{\Gamma(\sigma)}{y^\sigma} = \int_0^\infty e^{-yz} z^{\sigma-1}\, dz \tag{100}$$

the integral in Eq. 97 can be expressed as the series

$$\int_0^\infty \sqrt{z}\, G(z) e^{-k_a x_1 z}\, dz = \frac{G(0)\Gamma(3/2)}{(k_a x_1)^{3/2}} + \frac{G'(0)\Gamma(5/2)}{(k_a x_1)^{5/2}} + \cdots \tag{101}$$

Keeping only the first term in Eq. 101 gives a first order approximation for the branch cut integral,

$$u = - \frac{2\, e\, k_a^2 \sqrt{2j\pi}\, G(0)}{\bar{c}} \frac{\hat{\psi}_o\, e^{jk_a x_1}}{(k_a x_1)^{3/2}} \tag{102}$$

where

$$G(0) = \frac{\nu_b + \frac{\varepsilon}{\epsilon_o} \nu_c}{K^4 k_a^2}$$

and the radicals are evaluated at $k_1 = k_a$.

Eq. 102 represents an elastic wave propagating at the velocity of a uniform plane wave, $v_a = \frac{\omega}{k_a}$. The amplitude is proportional to $\frac{1}{(k_a x_1)^{3/2}}$. Waves with similar amplitude dependence have been shown to exist for purely elastic waves and also for purely electromagnetic waves at the surface of a dielectric [10,15,16]. These have been called lateral or head waves. The higher order terms in the Taylor expansion leads to waves with amplitude dependence $\frac{1}{(k_a x_1)^{5/2}}$, $\frac{1}{(k_a x_1)^{7/2}}$, ... For large x_1 these waves are negligible.

The evaluation of u around branch lines L_b and L_c and the evaluation of $\bar{E}^s$ and $\hat{\bar{E}}$ follow similar procedures. (From Eqs. 5, 6, and 12, $\bar{E}^s = - \bar{\nabla} \times \bar{n}\, \psi$ and $\hat{\bar{E}} = - \bar{\nabla} \times \bar{n}\, \hat{\psi}$.) The results are listed below.

Branch line L_a:

$$E_1^s = - \frac{j\, 2 \sqrt{2j\pi}\, \nu_b \nu_c}{K^2} \frac{\hat{\psi}_o\, e^{jk_a x_1}}{(k_a x_1)^{3/2}} \tag{103a}$$

$$E_2^s = \frac{jk_a}{\nu_b} E_1^s \tag{103b}$$

$$\hat{E}_2 = \frac{\epsilon}{\epsilon_o} E_2^s \tag{103c}$$

$$\hat{E}_1 = \frac{-\nu_c}{jk_a} \hat{E}_2 \tag{103d}$$

where the radicals are evaluated at $k_1 = k_a$.

Branch line L_b:

$$u = \frac{-2e\sqrt{2j\pi}\,\nu_a\nu_c k_b^3}{\overline{c}(K^2k_b^2 - \frac{\epsilon}{\epsilon_o}\nu_a\nu_c)^2} \; \frac{\hat{\psi}_o e^{jk_bx_1}}{(k_bx_1)^{3/2}} \tag{104a}$$

$$E_1^s = \frac{-j\,2\sqrt{2j\pi}\,\nu_a\nu_c k_b^2}{K^2k_b^2 - \frac{\epsilon}{\epsilon_o}\nu_a\nu_c} \; \frac{\hat{\psi}_o e^{jk_bx_1}}{(k_bx_1)^{3/2}} \tag{104b}$$

$$E_2^s = \frac{2\sqrt{2j\pi}\,\nu_c k_b^3\nu_a^2}{(K^2k_b^2 - \frac{\epsilon}{\epsilon_o}\nu_a\nu_c)^2} \; \frac{\hat{\psi}_o e^{jk_bx_1}}{(k_bx_1)^{3/2}} \tag{104c}$$

$$\hat{E}_2 = \frac{\epsilon}{\epsilon_o} E_2 \tag{104d}$$

$$\hat{E}_1 = \frac{-\nu_c}{jk_b} \hat{E}_2 \tag{104e}$$

where the radicals are evaluated at $k_1 = k_b$.

Branch line L_c:

$$u = \frac{-2e\sqrt{2j\pi}\,k_c^3}{\bar{c}(K^2k_c^2 - \nu_a\nu_b)}\;\frac{\hat{\psi}_o\,e^{jk_cx_1}}{(k_cx_1)^{3/2}} \tag{105a}$$

$$E_1^s = \frac{-j\,2\sqrt{2j\pi}\,\nu_a\nu_bk_c^2}{K^2k_c^2 - \nu_a\nu_b}\;\frac{\hat{\psi}_o\,e^{jk_cx_1}}{(k_cx_1)^{3/2}} \tag{105b}$$

$$E_2^s = \frac{jk_c}{\nu_b}E_1 \tag{105c}$$

$$\hat{E}_2 = \frac{\epsilon}{\epsilon_o}E_2 \tag{105d}$$

$$\hat{E}_1 = j\,2\sqrt{2j\pi}\,k_c^2\;\frac{\hat{\psi}_o\,e^{jk_cx_1}}{(k_cx_1)^{3/2}} \tag{105e}$$

where the radicals are evaluated at $k_1 = k_c$ and Eqs. 105d and 105e are the total external electric field, i.e., the source and reflected field.

The physical interpretation of the lateral waves is as follows:

Branch line L_a:

When the electromagnetic source field is incident on the piezoelectric free space interface, elastic waves are excited through the coupling of the electric and elastic fields at the interface. Many elastic wave-lengths from the source the elastic disturbance at the surface propagates as a surface wave with its associated electric fields, Eqs. 90 resulting from the pole of integral 86b, and a lateral wave, Eq. 102 calculated from branch line integral 91. The lateral wave propagates at the velocity of a uniform plane elastic wave. Coupled to the elastic lateral wave at the surface are the internal and external electric fields, Eqs. 103.

Branch line L_b:

Part of the incident electromagnetic wave is refracted across the interface. Many electromagnetic wavelengths from the source the electromagnetic disturbance at the surface is a lateral wave, Eqs. 104b and 104c, propagating at the velocity of a uniform electromagnetic plane wave in a dielectric, $v_b = \frac{\omega}{k_b}$. Coupled to the lateral wave at the surface are the elastic wave, Eq. 104a, and the external electromagnetic field, Eqs. 104d and 104e.

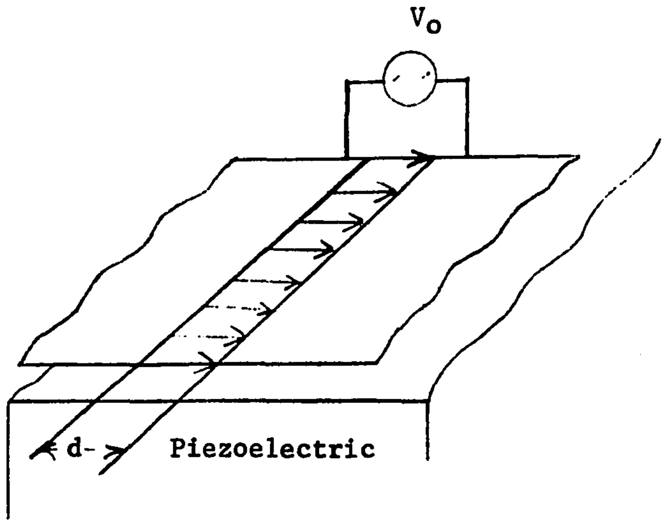
Branch line L_c:

Part of the electromagnetic wave is reflected. Many electromagnetic wavelengths from the source, the external field at the surface is given by the lateral wave, Eqs. 105d and 105e. The lateral wave propagates at the velocity of a uniform electromagnetic wave in free space, $v_c = \frac{\omega}{k_c}$. Coupled to this wave are the elastic wave, Eq. 105a, and the internal electromagnetic wave, Eqs. 105d and 105e.

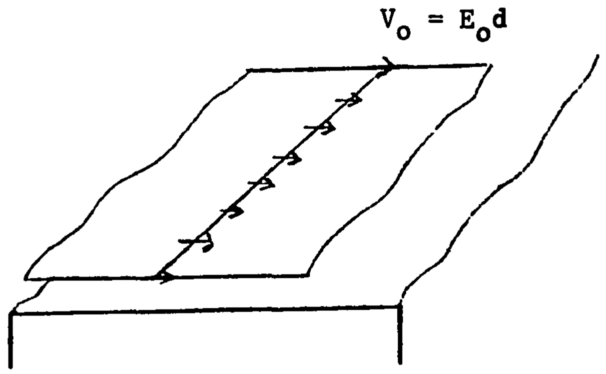
In closing this section we note that the elastic lateral wave and its associated electric fields satisfy the boundary conditions. Likewise, the lateral waves resulting from branch lines L_b and L_c and their associated elastic and electric fields satisfy the boundary conditions.

C. Electric Dipole Source in a Slotted Conductor

Another source of practical importance is shown in Figure 9a. This represents one pair of interdigital electrodes where the surface is



(a) Electrode pair



(b) Dipole approximation

Figure 9. External electromagnetic source in a slotted conductor

covered by a conductor except for the space between the electrodes. We can approximate the source by a line of dipoles in a slotted conductor as shown in Figure 9b. The field of such a source at $x_2 = -h$ is given by Eq. 82c with

$$\hat{\psi}_o = \frac{E_o d}{2\pi \nu_c} \tag{106}$$

The transmitted wave forms are given by Eqs. 2a and 2b. However, because of the conductor at $x_2 = -h$, the reflected wave form must be

$$\hat{\psi} = \int_{-\infty}^{\infty} C(k_1) \cosh \nu_c (x_2 + h) e^{jk_1 x_1} dk_1 \tag{107}$$

since this gives $\hat{E}_1 = 0$ at $x_2 = -h$.

Substituting the source, transmitted, and reflected fields into the boundary conditions, Eqs. 18, leads to the following integral solutions:

$$u = \frac{j e \hat{\psi}_o}{\bar{c}} \int_{-\infty}^{\infty} \frac{k_1 (1 + \tanh \nu_c h)}{D_m(k_1)} e^{-\nu_a x_2 + jk_1 x_1} dk_1 \tag{108a}$$

$$\psi = -\hat{\psi}_o \int_{-\infty}^{\infty} \frac{\nu_a (1 + \tanh \nu_c h)}{D_m(k_1)} e^{-\nu_b x_2 + jk_1 x_1} dk_1 \tag{108b}$$

$$\hat{\psi} = -\hat{\psi}_o \int_{-\infty}^{\infty} \left[\frac{\epsilon \nu_a (1 + \tanh \nu_c h)}{\epsilon_o D_m(k_1)} + \frac{1}{\nu_c} \right] \times \frac{\cosh \nu_c (x_2 + h)}{\cosh \nu_c h} e^{-\nu_c h + jk_1 x_1} dk_1 \tag{108c}$$

where

$$D_m(k_1) = K^2 k_1^2 - \nu_a [\nu_b + \frac{\varepsilon}{\varepsilon_o} \nu_c \tanh \nu_c h] \tag{108d}$$

The surface wave and lateral waves are evaluated by the method used in the previous chapter. With the source on the surface, h = 0 and the surface wave is

$$u = \frac{-2\pi e k_s \hat{\psi}_o e^{-\nu_a x_2 + jk_s x_1}}{\bar{c} D_m'(k_s)} \tag{109a}$$

$$\psi = \frac{-2\pi j \nu_a \hat{\psi}_o e^{-\nu_a x_2 + jk_s x_1}}{D_m'(k_s)} \tag{109b}$$

where

$$D_m'(k_s) = 2K^2 k_s - k_s \left(\frac{\nu_b}{\nu_a} + \frac{\nu_a}{\nu_b} \right) \tag{109c}$$

and the radicals are evaluated at $k_1 = k_s$.

For $h = x_2 = 0$ the lateral wave solutions are listed below:

Branch line L_a:

$$u = \frac{-e\sqrt{2j\pi}\,\nu_b}{\bar{c} K^4 k_a} \frac{\hat{\psi}_o e^{jk_a x_1}}{(k_a x_1)^{3/2}} \tag{110a}$$

$$E_1 = \frac{-j\sqrt{2j\pi}\,\nu_b}{K^2} \frac{\hat{\psi}_o e^{jk_a x_1}}{(k_a x_1)^{3/2}} \tag{110b}$$

$$E_2 = j \frac{k_a}{\nu_b} E_1 \tag{110c}$$

where the radicals are evaluated at $k_1 = k_a$.

Branch line L_b:

$$u = \frac{-e\sqrt{2j\pi}\ \nu_a}{\bar{c}\ K^4 k_b} \frac{\hat{\psi}_o e^{jk_b x_1}}{(k_b x_1)^{3/2}} \tag{111a}$$

$$E_1 = \frac{-j\sqrt{2j\pi}\ \nu_a}{K^2} \frac{\hat{\psi}_o e^{jk_b x_1}}{(k_b x_1)^{3/2}} \tag{111b}$$

$$E_2 = \frac{\sqrt{2j\pi}\ \nu_a^2}{K^4 k_b} \frac{\hat{\psi}_o e^{jk_b x_1}}{(k_b x_1)^{3/2}} \tag{111c}$$

where the radicals are evaluated at $k_1 = k_b$.

The amplitude of the elastic lateral wave relative to the amplitude of the surface wave must be considered in the design of Bleustein-Gulyaev wave devices. If the surface wave detector is not far enough from the source, the signal from the elastic lateral wave may be larger than the signal from the surface wave. Ratios of elastic lateral wave amplitude to surface wave amplitude at a distance of 100 elastic wavelengths from the source for a conducting surface are given below. The ratios are calculated from Eqs. 90a and 110a using material parameters given in Reference [17].

For ZnO where the coupling constant $K^2 = 0.1$,

$$R = 0.025 \ . \tag{112a}$$

For CdS where $K^2 = 0.0355$,

$$R = 0.57 \ . \tag{112b}$$

For CdSe where $K^2 = 0.0173$,

$$R = 4.9 . \tag{112c}$$

Note that as the coupling constant decreases the ratio increases.

If the surface is not covered by a conductor, the ratios are greater by a factor of approximately $(1 + \frac{\varepsilon}{\varepsilon_o})^3$. As an example, Eqs. 90a and 102 give an amplitude ratio for ZnO of

$$R = 21 . \tag{112d}$$

V. EXCITATION OF BLEUSTEIN-GULYAEV WAVES ON A CYLINDRICAL SURFACE

A. Solid Cylinder

Surface waves on a solid piezoelectric cylinder may be excited by an electromagnetic or elastic source in the cylinder or an electromagnetic source outside the cylinder. Here we will derive the solutions for an external electric dipole line at $r = h$, $\theta = 0$ as shown in Figure 10. The dipole points in the θ direction and the line of dipoles is parallel to the x_3 axis (not shown).

For a dipole of strength $E_o d$ the electric potential is

$$\hat{\psi}^o = \hat{\psi}_o H(k_c|\bar{r} - \bar{h}|) \sin \xi \tag{113a}$$

where

$$\hat{\psi}_o = \frac{-jk_c E_o d}{4} , \tag{113b}$$

the angle ξ is shown in Figure 10, and the Hankel function is the first kind.

Eq. 113 may be expressed as a series of Bessel functions with origin at $r = 0$ by Graf's addition theorem,

$$\hat{\psi}^o = \hat{\psi}_o \sum_{n=-\infty}^{\infty} H_{n+1}(k_c h) J_n(k_c r) \cos(n\theta), \; r < h \tag{114}$$

With the relations $J_{-n}(x) = (-1)^n J_n(x)$, $H_{-n}(x) = (-1)^n H_n(x)$, and $H_n'(x) = - H_{n+1}(x) + \frac{n}{x} H_n(x)$, Eq. 114 may be written as

$$\hat{\psi}^o = - \hat{\psi}_o \sum_{n=0}^{\infty} \epsilon_n H_n'(k_c h) J_n(k_c r) \cos(n\theta) \tag{115}$$

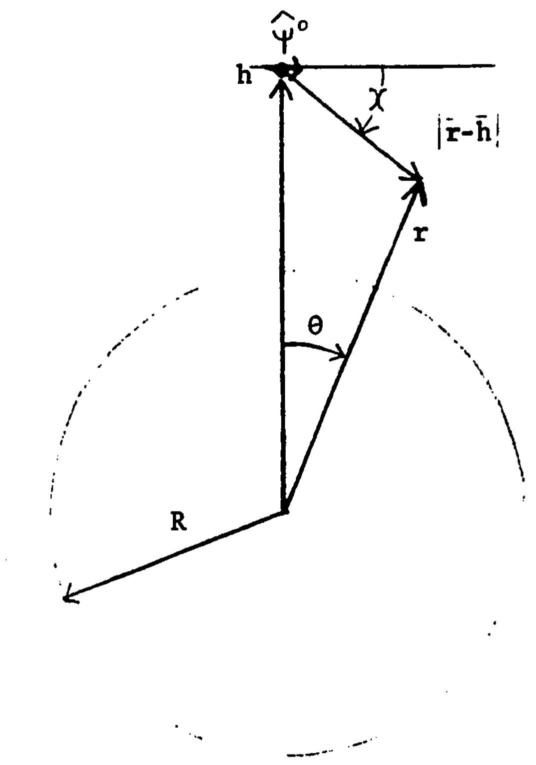


Figure 10. Dipole line source above piezoelectric cylinder

where

$$\epsilon_n = \begin{cases} 1 & n = 0 \\ 2 & n \neq 0 \end{cases} .$$

The waveforms of the scattered fields are

$$u = \sum_{n=0}^{\infty} A(n) J_n(k_a r) \sin(n\theta) \tag{116a}$$

$$\psi = \sum_{n=0}^{\infty} B(n) J_n(k_b r) \cos(n\theta) \tag{116b}$$

$$\hat{\psi} = \sum_{n=0}^{\infty} C(n) H_n(k_c r) \cos(n\theta) \tag{116c}$$

Substituting Eqs. 115 and 116 into the boundary conditions given in Eqs. 65 leads to the following series solutions:

$$u = \sum_{n=0}^{\infty} \epsilon_n \frac{A_1(n)}{D(n)} J_n(k_a r) \sin(n\theta) \tag{117a}$$

$$\psi = \sum_{n=0}^{\infty} \epsilon_n \frac{B_1(n)}{D(n)} J_n(k_b r) \cos(n\theta) \tag{117b}$$

$$\hat{\psi} = \sum_{n=0}^{\infty} \epsilon_n \frac{C_1(n)}{D(n)} H_n(k_c r) \cos(n\theta) \tag{117c}$$

where

$$A_1 = \frac{-2\, j\, e\, \hat{\psi}_o H_n'(k_c h) n}{\pi\, \bar{c}\, H_n(x_c) J_n(x_a)} \tag{117d}$$

$$B_1 = \frac{-2\, j\, \hat{\psi}_o H_n'(k_c h) x_a J_n'(x_a)}{\pi\, J_n(x_b) J_n(x_c)} \tag{117e}$$

$$C_1 = \frac{\hat{\psi}_o H'_n(k_c h) J_n(x_c)}{H_n(x_c)} \left\{ K^2 n^2 - \frac{x_a J'_n(x_a)}{J_n(x_a)} \left[\frac{x_b J'_n(x_b)}{J_n(x_b)} - \frac{\epsilon \, x_c J'_n(x_c)}{\epsilon_o J_n(x_c)} \right] \right\} \tag{117f}$$

and

$$D(n) = K^2 n^2 - \frac{x_a J'_n(x_a)}{J_n(x_a)} \left[\frac{x_b J'_n(x_b)}{J_n(x_b)} - \frac{\epsilon \, x_c H'_n(x_c)}{\epsilon_o H_n(x_c)} \right] \tag{117g}$$

Note that equating Eq. 117g to zero gives the propagation constant equation, Eq. 66c. To calculate expressions pertaining to the surface wave we must change the series solutions to integral solutions and evaluate the residues at D(n) = 0. This may be accomplished through either the Watson transformation [18] or Poisson's summation formula [19]. The form of Poisson's summation formula used here is [20]

$$\sum_{n=0}^{\infty} \epsilon_n f(n) = 2 \sum_{m=0}^{\infty} \epsilon_m \int_0^{\infty} f(\tau) \cos(2\pi m \tau) d\tau \tag{118}$$

With this Eq. 116a becomes

$$u = 2 \sum_{m=0}^{\infty} \epsilon_m \int_0^{\infty} \frac{A_1(\tau)}{D(\tau)} J_\tau(k_a \tau) \sin(\tau\theta) \cos(2\pi m \tau) d\tau \tag{119}$$

To find the residues of these integrals we close the path of integration in the first or fourth quadrant as shown in Figure 11. For integral 119 to vanish on the paths at infinity the integrand multiplied by τ must vanish as τ approaches infinity. This is shown by using the asymptotic expression for Bessel functions of large order.

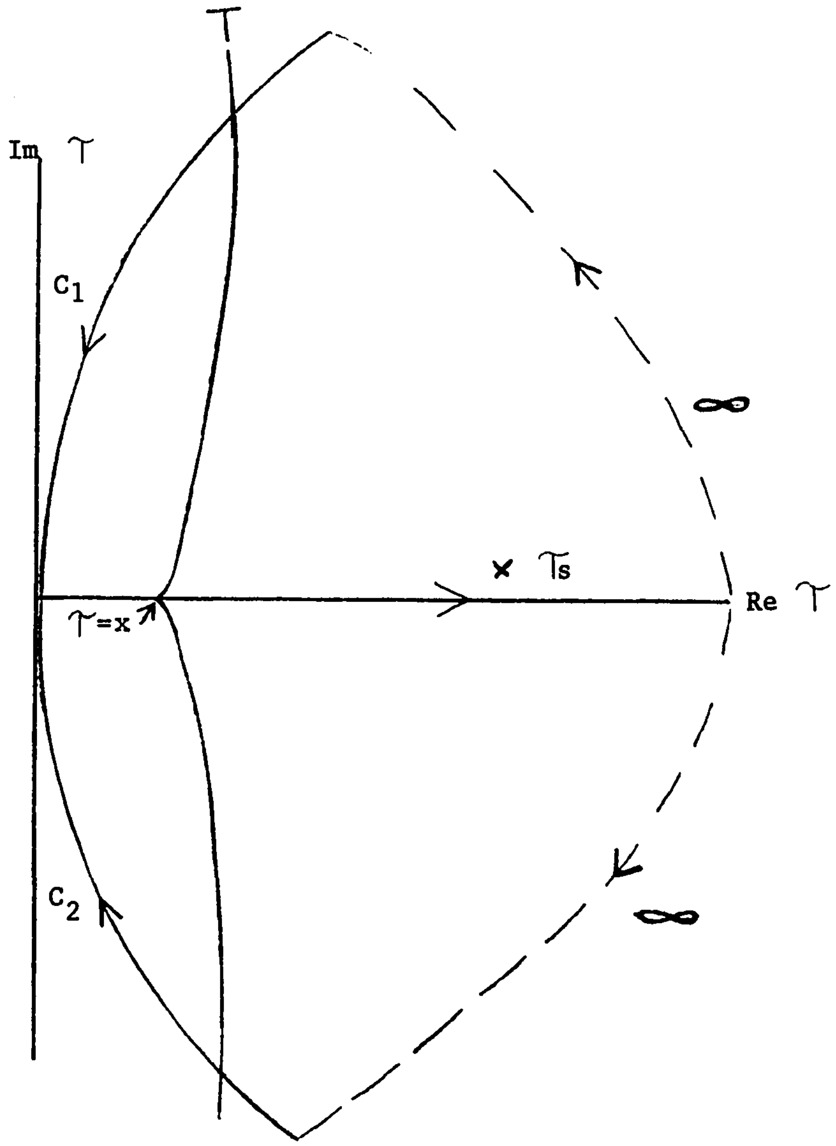


Figure 11. Paths of integration in complex τ plane

In the region to the right of the line labelled T in Figure 11, the asymptotic functions have the exponential dependence [21]

$$H_\tau(x) \propto H'_\tau(x) \propto e^{-\tau + \tau \ln 2\tau - \tau \ln x} \tag{120a}$$

$$J_\tau(x) \propto e^{\tau - \tau \ln 2\tau + \tau \ln x} \quad . \tag{120b}$$

The Bessel function factor in integral 119 will have the asymptotic form

$$\frac{H'_\tau(k_c h) J_\tau(k_a r)}{H_\tau(x_c) J_\tau(x_a)} \to e^{-\tau \ln \frac{h}{r}} \tag{121}$$

Since $\frac{h}{r} > 1$, $\tau e^{-\tau \ln \frac{h}{r}} \to 0$ as τ approaches infinity and integral 119 vanishes on the paths at infinity. The choice of the path of integration, therefore, depends on the behavior of the trigonometric functions in the first and fourth quadrants. Writing the trigonometric functions in exponential form and choosing the path of integration that keeps the integrands bounded leads to the residue series

$$u = 2\pi j \sum_{p=1}^{\infty} \frac{A_1(\tau_p)}{D'(\tau_p)} \left[\sum_{m=0}^{\infty} e^{j \tau_p (\theta + 2\pi m)} - \sum_{m=0}^{\infty} e^{-j \tau_p (\theta - 2\pi m)} \right] \tag{122}$$

plus integrals over paths C_1 and C_2. The summation over the roots of $D(\tau)$ where τ is in the first quadrant includes the surface wave which is the largest root, say τ_s, the faster "whispering gallery waves", plus other roots such as those giving the "creeping waves" of the geometrically diffracted electromagnetic waves.

The physical interpretation of solution 122 is that the series

$$\sum_{m=0}^{\infty} e^{j \tau_p (\theta + 2\pi m)}$$

represents waves traveling in the $+\theta$ direction (with $e^{-j\omega t}$ time dependence). If we restrict $-2\pi < \theta < 2\pi$, the wave has made "m" circumnavigations of the cylinder. Likewise the series

$$\sum_{m=0}^{\infty} e^{-j \tau_p (\theta + 2\pi m)}$$

represents waves traveling in the $-\theta$ direction which have made "m" circumnavigations in the $-\theta$ direction. Since τ_p is complex the wave amplitude will decrease in the direction of propagation. Furthermore, path C_2 must not encircle any poles in the fourth quadrant (if there are any) since this would result in waves which increase in magnitude in the direction of propagation.

The reason for converting the original source Eq. 114 to a series summed over only the nonnegative integers is now apparent. If the solutions, Eqs. 117, were summed over negative and positive integers, the Poisson summation formula would convert the series to integrals which are integrated from $-\infty$ to ∞ [19]. Closing the path of integration in the upper or lower half plane may include poles in the second or third quadrants where Re $\tau < 0$. Since $J_\tau(k_a r) \to \infty$ as $r \to 0$ if Re $\tau < 0$ and τ is not an integer, the residues resulting from these poles would give physically unacceptable solutions.

The "background integrals" over C_1 and C_2 will not be evaluated here since we are mainly interested in the surface wave resulting from the residue. Background integrals also appear in the scattering of acoustic waves from elastic cylinders [12].

The electric potentials, Eqs. 117b and 117c, may be evaluated in a similar manner.

B. Flat Surface Limit

When the radius of the cylinder approaches infinity, the cylindrical surface wave solution must approach the plane surface wave solution. This is shown in the following manner. From Eq. 122 the surface wave traveling in the $+\theta$ direction is with $m=0$,

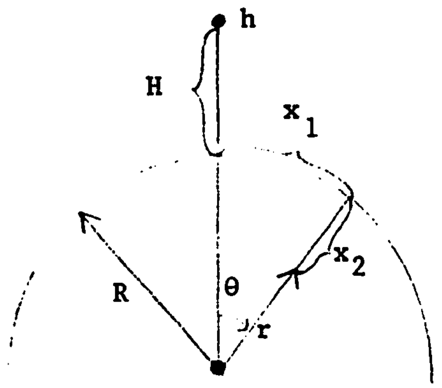
$$u = \frac{-j\,4\,e\,\hat{\psi}_o}{\bar{c}} \; \frac{H'_{\tau_s}(k_c h)\tau_s J_{\tau_s}(k_a r)}{H_{\tau_s}(x_c) J_{\tau_s}(x_a) D'(\tau_s)} \, e^{j\tau_s \theta} \tag{123}$$

Referring to Figure 12 we make the substitutions

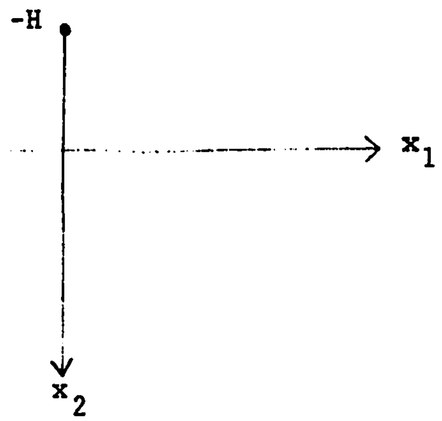
$$H = h-R, \; x_2 = R-r, \; x_1 = r\theta, \text{ and } \tau = kR \tag{124}$$

in Eq. 123 and take the limit $R \to \infty$ while keeping H, x_2, and x_1 constant. Keeping only the first terms in Eqs. 148 of the Appendix leads to large radius approximations

$$\frac{J_{\tau_s}(k_a r)}{J_{\tau_s}(k_a R)} \to e^{-\sqrt{k_s^2 - k_a^2}\, x_2} \tag{125a}$$



(a) Cylindrical Coordinates



(b) Corresponding cartesian coordinates when radius R approaches infinity

Figure 12. Flat surface limit

$$\frac{H'_{\tau_s}(k_c h)}{H_{\tau_s}(k_c R)} \rightarrow \frac{-\sqrt{k_s^2-k_c^2}}{k_c} e^{-\sqrt{k_s^2-k_c^2}\, H} \tag{125b}$$

$$D(\tau) \rightarrow [K^2k^2 - \sqrt{k^2-k_a^2}\,(\sqrt{k^2-k_b^2} + \frac{\varepsilon}{\varepsilon_o}\sqrt{k^2-k_c^2})]R^2 \tag{125c}$$

Substituting Eqs. 124 and 125 into Eq. 123 leads to

$$u = \frac{j\,4\,e\,\hat{\psi}_o}{\bar{c}\,k_c} \left. \frac{\sqrt{k_s^2-k_c^2}\,k_s\,e^{-\sqrt{k_s^2-k_c^2}\,H - \sqrt{k_s^2-k_a^2}\,x_2 + jk_s x_1}}{\frac{d}{dk}[K^2k^2 - \sqrt{k^2-k_a^2}\,(\sqrt{k^2-k_b^2} + \frac{\varepsilon}{\varepsilon_o}\sqrt{k^2-k_c^2})]} \right|_{k = k_s} \tag{126}$$

The solution for a flat surface is given by Eq. 90a. With the expressions for $\hat{\psi}_o$, Eqs. 85 and 113b, Eq. 90a and Eq. 126 are identical.

VI. BONUS SECTION: ELASTIC-ELECTROMAGNETIC ENERGY CONVERSION

A. Conducting Surface

In the previous sections we calculated the fields excited by dipole sources. In the flat surface case we constructed the fields from uniform and inhomogeneous plane waves. Through the coupling of the fields at the surface, part of the energy of an incident elastic wave is converted to electromagnetic energy and part of the energy of an incident electromagnetic wave is converted to elastic energy. Here we will demonstrate that, in certain cases of an incident uniform plane wave, the conversion is complete.

The first case to be examined is an incident uniform elastic plane wave on a metallized surface. Figure 13 shows the geometry where, for this case, the external field is zero. The reflection coefficients and wave forms for plane waves have been derived in Section IV A, Eqs. 84. With the substitutions

$$k_1 = k_a \sin\theta = k_b \sin\theta_e \tag{127a}$$

$$\sqrt{k_a^2 - k_1^2} = k_a \cos\theta \tag{127b}$$

$$\sqrt{k_b^2 - k_1^2} = \sqrt{k_b^2 - k_a^2 \sin^2\theta} \tag{127c}$$

where θ is the angle of incidence, Eqs. 84 give

$$u^o = u_o\, e^{jk_a \sin\theta\, x_1 - jk_a \cos\theta\, x_2} \tag{128a}$$

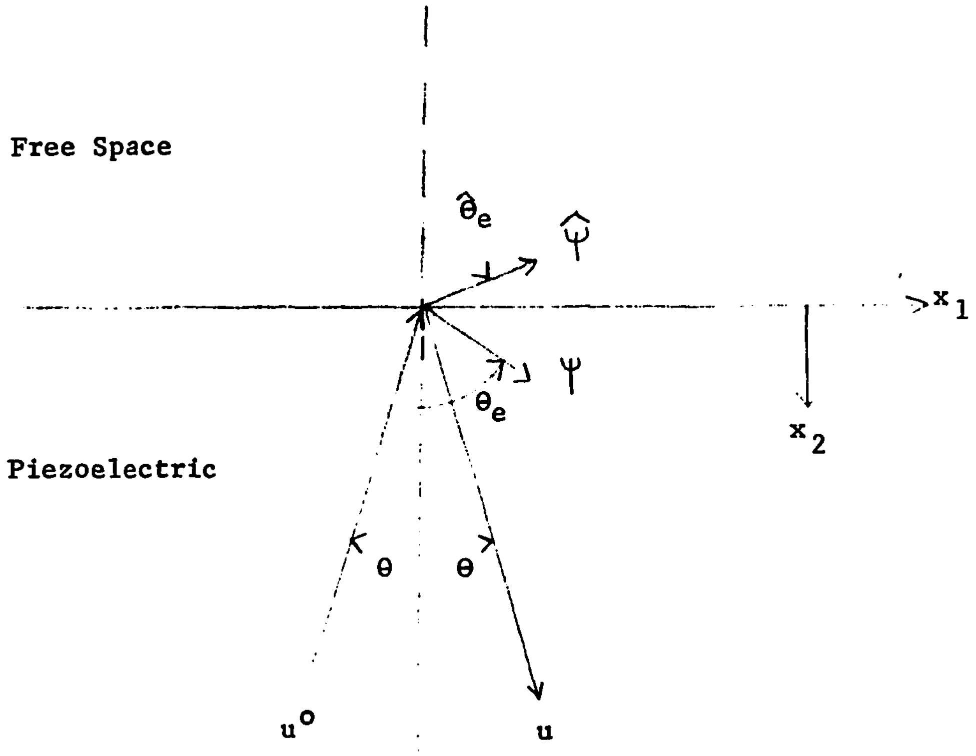


Figure 13. Incident, reflected, and transmitted fields

$$u = A e^{jk_a \sin\theta\, x_1 + jk_a \cos\theta\, x_2} \tag{128b}$$

$$\psi = B e^{jk_b \sin\theta_e\, x_1 + jk_b \cos\theta_e\, x_2} \tag{128c}$$

where

$$\frac{A}{u_o} = \frac{K^2 \sin^2\theta - \cos\theta \sqrt{\frac{k_b^2}{k_a^2} - \sin^2\theta}}{D_1} \tag{128d}$$

$$\frac{B}{u_o} = \frac{2\frac{e}{\epsilon}\cos\theta \sin\theta}{D_1} \tag{128e}$$

and

$$D_1 = -K^2 \sin^2\theta - \cos\theta \sqrt{\frac{k_b^2}{k_a^2} - \sin^2\theta} \tag{128f}$$

Eq. 127a, which matches the propagation constants along the surface, shows that a uniform plane elastic or electromagnetic wave will not excite a piezoelectric surface wave. The surface wave propagation constant is larger than the uniform plane wave propagation constant, $k_1 > k_a \gg k_b$, which gives imaginary angle of incidence in Eq. 127a.

Examination of the elastic reflection coefficient 128d shows that there is a critical value of θ where the reflection coefficient is zero. At this angle the incident elastic wave is completely converted at the surface to a reflected electromagnetic wave. This somewhat surprising phenomenon may not have been discovered previous to this study since the electrostatic approximation is often used to simplify piezoelectric wave problems. If the electrostatic approximation is used here, the reflection coefficient is

$$\frac{A}{u_o} = \frac{k^2 \sin\theta + j\cos\theta}{-k^2 \sin\theta + j\cos\theta} \tag{129}$$

which is never zero.

To solve for the critical angle, we note that for the radical in the numerator of Eq. 128d to be real, $\sin^2\theta < \frac{k_b{}^2}{k_a{}^2}$. Since $\frac{k_b{}^2}{k_a{}^2} \approx 10^{-8}$ to 10^{-10}, small angle approximation will be very accurate. The numerator of Eq. 128d is then

$$N_1 = K^2\theta^2 - \sqrt{\frac{k_b^2}{k_a^2} - \theta^2} = 0 \tag{130}$$

Squaring Eq. 130, applying the quadratic formula, and expanding the resultant radical leads to the critical angle,

$$\theta_c \approx \frac{k_b}{k_a}\left(1 - \frac{K^4}{2}\frac{k_b^2}{k_a^2}\right) \tag{131}$$

The extremely narrow range where transduction is significant can be demonstrated by plotting the ratio of the x_2 component of the reflected electromagnetic Poynting vector to the x_2 component of the incident elastic Poynting vector as a function of θ. The Poynting vector for the incident elastic wave is

$$P_2 = \frac{\omega\,\bar{c}}{2} k_a \cos\theta\, |u_o|^2 \tag{132}$$

and for the reflected electromagnetic wave

$$S_2 = \frac{\omega\,\varepsilon}{2} k_b \cos\theta_e\, |B|^2 \tag{133}$$

which, with Eqs. 127, 128e, and the small angle approximations, lead to

$$\frac{S_2}{P_2} = \frac{4\, K^2\theta^2 \sqrt{\frac{k_b^2}{k_a^2} - \theta^2}}{\left(K^2\theta^2 + \sqrt{\frac{k_b^2}{k_a^2} - \theta^2}\right)^2} \quad . \tag{134}$$

Since $\theta_c \approx \frac{k_b}{k_a}$ it is helpful to plot Eq. 134 as a function of δ where

$$\theta = \frac{k_b}{k_a} - \delta \tag{135}$$

This is shown in Figure 14 for the piezoelectric ceramic PZT-5A with $K^2 = 0.47$ and $\frac{k_b}{k_a} = 2.287 \times 10^{-4}$. At $\theta = \theta_c$ the power ratio is unity and at $\theta = \frac{k_b}{k_a}$ and at $\theta = 0$ the power ratio is zero. Significant transduction occurs over a range of only 10^{-7} radians.

At the critical angle the reflection angle of the electromagnetic wave is given by Eqs. 127a and 131,

$$\sin\theta_e = \frac{k_a}{k_b}\sin\theta_c \approx 1 - \frac{K^4 k_b^2}{2\, k_a^2} \tag{136}$$

and the electromagnetic wave propagates almost parallel to the surface.

The transduction is reversible. That is, if an electromagnetic wave is incident at a critical angle given by Eq. 136, the only reflected wave will be an elastic wave propagating at an angle almost normal to the surface given by Eq. 131.

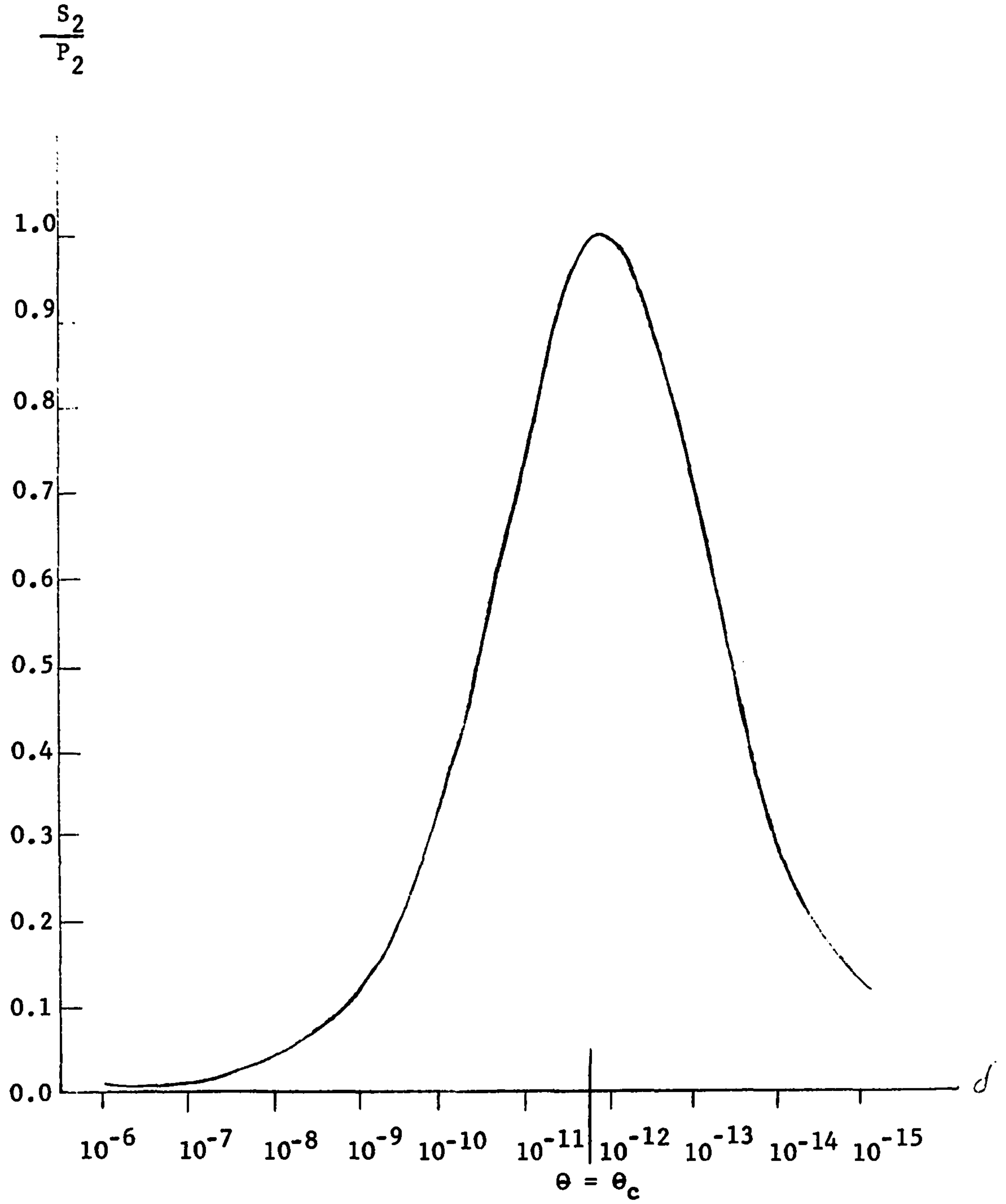


Figure 14. Power ratio

B. Free Surface

Another case where complete transduction occurs is an elastic wave incident on an interface of a piezoelectric medium and a fluid of permittivity ϵ_o. The analysis is similar to the previous case except there is an external electromagnetic wave as shown in Figure 13,

$$\hat{\psi} = C\, e^{jk_c \sin \hat{\theta}_e x_1 - jk_c \cos \hat{\theta}_e x_2} \tag{137}$$

Eqs. 84 and the relations $k_1 = k_c \sin \hat{\theta}_e = k_a \sin \theta$ lead to the ratios

$$\frac{A}{u_o} = \frac{1}{D_2} \left[K^2 \sin^2 \theta - \cos \theta \left(\sqrt{\frac{k_b^2}{k_a^2} - \sin^2 \theta} + \frac{\epsilon}{\epsilon_o} \sqrt{\frac{k_c^2}{k_a^2} - \sin^2 \theta} \right) \right] \tag{138a}$$

$$\frac{B}{u_o} = \frac{2\, e \sin \theta \cos \theta}{\epsilon\, D_2} \tag{138b}$$

$$\frac{C}{B} = \frac{\epsilon}{\epsilon_o} \tag{138c}$$

where

$$D_2 = - K^2 \sin^2 \theta - \cos \theta \left(\sqrt{\frac{k_b^2}{k_a^2} - \sin^2 \theta} + \frac{\epsilon}{\epsilon_o} \sqrt{\frac{k_c^2}{k_a^2} - \sin^2 \theta} \right) \tag{138d}$$

Total energy conversion occurs if the numerator of Eq. 138a is zero. With small angle approximations setting the numerator equal to zero gives

$$N_2 = K^2\theta^2 - \left(\sqrt{\frac{k_b^2}{k_a^2} - \theta^2} + \frac{\epsilon}{\epsilon_o} \sqrt{\frac{k_c^2}{k_a^2} - \theta^2} \right) = 0 . \tag{139}$$

Examination of Eq. 139 shows that a critical angle exists only for certain ranges of ϵ_o. These ranges and approximate values for the critical angle are derived in the following manner. At the critical angle, $N_2 = 0$ and at $\theta = 0$, $N_2 < 0$. Since N_2 increases as θ increases, the maximum value of N_2 occurs at the largest permissible θ, either $\theta = \frac{k_b}{k_a}$ or $\theta = \frac{k_c}{k_a}$. Therefore, a critical angle occurs only for ranges of ϵ_o where the maximum of $N_2 \geq 0$. When $\epsilon_o < \epsilon$, substituting $\theta = \frac{k_c}{k_a}$ into Eq. 139 leads to

$$N_2 = K^2 \frac{k_c^2}{k_a^2} - \sqrt{\frac{k_b^2}{k_a^2} - \frac{k_c^2}{k_a^2}} = 0. \tag{140}$$

Recalling that $k_b^2 = \omega^2 u_o \epsilon$ and $k_c^2 = \omega^2 u_o \epsilon_o$ and solving for $\frac{\epsilon_o}{\epsilon}$ gives

$$\frac{\epsilon_o}{\epsilon} \approx 1 - K^4 \frac{k_b^2}{k_a^2} . \tag{141}$$

When $\epsilon_o > \epsilon$, substituting $\theta = \frac{k_b}{k_a}$ into Eq. 139 leads to

$$N_2 = K^2 \frac{k_b^2}{k_a^2} - \frac{\epsilon}{\epsilon_o} \sqrt{\frac{k_c^2}{k_a^2} - \frac{k_b^2}{k_a^2}} = 0 \tag{142}$$

Solving for $\frac{\epsilon_o}{\epsilon}$ gives

$$\frac{\epsilon_o}{\epsilon} \approx 1 + K^4 \frac{k_b^2}{k_a^2} \tag{143a}$$

and

$$\frac{\epsilon_o}{\epsilon} \approx \frac{k_a^2}{K^4 k_b^2} . \tag{143b}$$

From Eqs. 141 and 143 the ranges of ϵ_o where complete transduction can exist are

$$\epsilon(1 - K^4 \frac{k_b^2}{k_a^2}) < \epsilon_o < \epsilon(1 + K^4 \frac{k_b^2}{k_a^2}) \tag{144a}$$

and

$$\epsilon_o > \frac{\epsilon\, k_a^2}{K^4 k_b^2} \tag{144b}$$

In the range specified by Eq. 144a, $\epsilon_o \approx \epsilon$ and Eq. 139 becomes

$$N_2 \approx K^2\theta^2 - 2\sqrt{\frac{k_b^2}{k_a^2} - \theta^2} = 0 . \tag{145}$$

Comparing this with Eqs. 130 and 131 gives the critical angle,

$$\theta_c \approx \frac{k_b}{k_a}(1 - \frac{K^4}{8} \frac{k_b^2}{k_a^2}) . \tag{146}$$

At this angle the incident elastic wave is converted to a reflected and a transmitted electromagnetic wave.

In the range specified by Eq. 144b, $\epsilon_o \gg \epsilon$ and Eq. 139 becomes

$$N_2 \approx K^2\theta^2 - \sqrt{\frac{k_b^2}{k_a^2} - \theta^2} = 0 . \tag{147}$$

This is identical to Eq. 130 so the critical angle is given by Eq. 131.

In the other two cases of electromagnetic waves incident on a free space interface from either side, transduction occurs for all angles but is never complete since there is always a transmitted electromagnetic wave.

VII. SUMMARY

This thesis analyzes a number of related problems involving the excitation and propagation of Bleustein-Gulyaev waves. The analyses should prove helpful in the design of Bleustein-Gulyaev wave devices employing piezoelectrics which are transversely isotropic. The results of each section are summarized below.

Section II compares the electrodynamic solution of the Bleustein-Gulyaev wave on a flat surface to the electrostatic solution. The electrostatic approximation is found to give a slightly faster propagation velocity than the true velocity. However, the error is much less than the accuracy to which the material parameters can be measured. The decoupling of the elastic and electromagnetic waves in an unbounded piezoelectric is also examined.

Section III derives the solutions for the propagation of Bleustein-Gulyaev waves around cylindrical surfaces. It is found that the propagation constant of a surface wave traveling around a solid cylinder is complex. Therefore, the amplitude of the surface wave decreases in the direction of propagation as electromagnetic energy radiates away from the cylinder. If the cylinder is covered by an electric conductor there is no external field and the propagation constant is real. When a surface wave propagates around a cylindrical cavity, both elastic and electromagnetic energy radiate away from the surface into the piezoelectric. Large radii approximations for the propagation constant demonstrate that the curvature of a solid cylinder increases the phase velocity while the curvature of a cylindrical cavity decreases the phase velocity. These approximations also show that the electrostatic approximation is adequate for large radii.

Section IV examines the excitation of Bleustein-Gulyaev waves on a flat surface by an electric dipole line source in free space and in a slotted conductor. Exact expressions for the surface wave and approximate expressions for the electromagnetic and elastic lateral waves are derived. It is demonstrated that even at many wavelengths from the source the elastic lateral wave may be larger than the surface wave.

Section V derives integral solutions for the excitation of Bleustein-Gulyaev waves by an electric dipole line source above a solid cylinder. The residues of the integrals result in expressions representing the surface wave. These expressions are shown to reduce to the flat surface wave expressions as the radius of the cylinder approaches infinity.

Section VI analyzes the reflection of uniform plane elastic and electromagnetic waves from flat surfaces. If the surface is covered by an electric conductor, it is demonstrated that there exists a critical angle of incidence at which an incident elastic wave is completely converted to reflected elastic wave. Furthermore, for a free space interface there exists a critical angle of incidence only for certain ranges of the free space permittivity. At this critical angle an incident elastic wave is completely converted to reflected and transmitted electromagnetic waves.

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X. APPENDIX

Approximations for Bessel functions of the first and second kind for large order and argument are given in references [9,14,22]. In the region Re n ≫ 1 and $|n - x| > |n|^{1/3}$ keeping only the terms of order $\frac{1}{n}$ and $\frac{1}{x}$ in the series expansions leads to

$$J_n(x) = \frac{e^{f(n,x)}}{\sqrt{2\pi}\,(n^2 - x^2)^{1/4}}\,[1 - S] \tag{148a}$$

$$N_n(x) = \frac{-\,e^{-\,f(n,x)}}{\sqrt{\frac{\pi}{2}}\,(n^2 - x^2)^{1/4}}\,[1 + S] \tag{148b}$$

$$x\,J_n'(x) = \frac{(n^2 - x^2)^{1/4}\,e^{f(n,x)}}{\sqrt{2\pi}}\,[1 - T] \tag{148c}$$

$$x\,N_n'(x) = \frac{(n^2 - x^2)^{1/4}\,e^{-\,f(n,x)}}{\sqrt{\frac{\pi}{2}}}\,[1 + T] \tag{148d}$$

where

$$S = \frac{\frac{1}{12}\,n^2 + \frac{1}{8}\,x^2}{(n^2 - x^2)^{3/2}} \tag{148e}$$

$$T = \frac{\frac{1}{12}\,n^2 - \frac{3}{8}\,x^2}{(n^2 - x^2)^{3/2}} \tag{148f}$$

With these expressions the combination of Bessel functions found in the wave propagation equations of Section III are

$$\frac{x\,J_n'(x)}{J_n(x)} \approx (n^2 - x^2)^{1/2}\,[1 + (S-T)] \tag{149a}$$

and, with $H_n^{(1)}(x) = J_n(x) + j\,N_n(x)$

$$\frac{x\,H_n'(x)}{H_n(x)} \approx -\,(n^2 - x^2)^{1/2}\,[1 - (S-T) - j\,e^{2f(n,x)}] \tag{149b}$$

The method used to find explicit expressions for the propagation constants is demonstrated here for the propagation constant equation for a solid cylinder with a free space interface, Eq. 66c,

$$n^2K^2 = \frac{x_a J_n'(x_a)}{J_n(x_a)}\left[\frac{x_b J_n'(x_b)}{J_n(x_b)} - \frac{\varepsilon}{\varepsilon_o}\,\frac{x_c H_n'(x_c)}{H_n(x_c)}\right] \tag{150}$$

Substituting Eqs. 149 into Eq. 150 and using the approximations $\sqrt{n^2 - x_b^2} \approx n$ and $\sqrt{n^2 - x_c^2} \approx n$ leads to

$$n\,K^2 \approx (n^2 - x_a^2)^{1/2}\,\Big\{\,(1 + \frac{\varepsilon}{\varepsilon_o})\,[1 + \frac{x_a^2}{2(n^2 - x_a^2)^{3/2}}\,] - j\,\frac{\varepsilon}{\varepsilon_o}\,e^{2f(n,x_c)}\,\Big\} \tag{151}$$

Squaring Eq. 151 and keeping only the terms of order $\frac{1}{n}$, $\frac{1}{x}$, and $e^{2f(n,x_c)}$ gives

$$n^2K^4 \approx (n^2 - x_a^2) \quad (1 + \frac{\epsilon}{\epsilon_o})^2 [1 + \frac{x_a^2}{(n^2 - x_a^2)^{3/2}}$$

$$- \frac{j\,2}{(1 + \frac{\epsilon_o}{\epsilon})} e^{2f(n,x_c)}] \tag{152}$$

Manipulating Eq. 152 and using the relations $n = k\,R$, $x_a = k_a R$ and $x_c = k_c R$ gives

$$k^2 \approx \frac{k_a^2}{k - \frac{K^4}{(1 + \frac{\epsilon}{\epsilon_o})^2}} [1 - \frac{1}{(k^2 - k_a^2)^{1/2} R}$$

$$+ j \frac{2(k^2 - k_a^2)\, e^{2f(k,k_c)R}}{k_a^2 (1 + \frac{\epsilon_o}{\epsilon})}] \; . \tag{153}$$

For very large R,

$$k^2 \approx \frac{k_a^2}{1 - \frac{K^4}{(1 + \frac{\epsilon}{\epsilon_o})^2}} = k_s^2 \tag{154}$$

which is the propagation constant for a flat surface, Eq. 39c. Since for large R the second and third terms in the brackets of Eq. 153 are much less than unity, Eq. 154 can be substituted into these terms with negligible error. Taking the square root of Eq. 153 then leads to the approximation

$$k \approx k_s \left[1 - \frac{1}{2(k_s - k_a)^{1/2}R}\right] + j \frac{(k_s^2 - k_a^2)k_s}{k_a^2 \left(1 + \frac{\epsilon_o}{\epsilon}\right)} e^{2f(k_s,k_c)R} \tag{155}$$

which is Eq. 78a.